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ABSTRACT

This document updates Texas Education Agency Bulletin 730 printed in 1973, and the guidelines for mathematics programs provided are designed to be used by school districts in designing or updating local curriculum and instructional programs. The Introduction contains sections on Program Goals, Using the Program Guide, Instructional Aids, and Evaluation. A section titled Overview: Program Matrix provides a chart of the scope and sequence of the following topics in grades six through eight: Number, Numeration, and Place Value; Operations and Properties; Problem Solving; Measurement; Geometry; Graphing, Relations, Functions; Probability and Statistics; and Computer Literacy. The bulk of the document consists of individual sections for each grade level, subdivided into unit titles with objectives, suggested teaching activities, and resources/comments. Two appendices contain eight National Council of Teachers of Mathematics recommendations from "An Agenda for Action," and a correlation of TABS Objectives. (MP)

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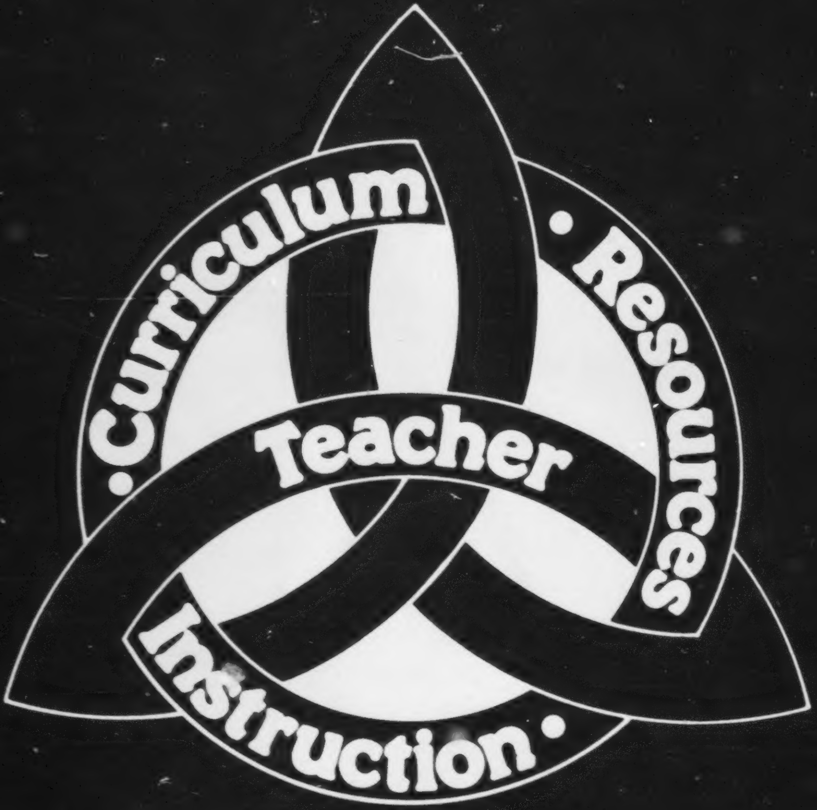

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MATHEMATICS
Levels: Six Through Eight

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Foreword

This publication and a companion publication, ***Mathematics, Levels: Kindergarten Through Five,*** are revisions of Texas Education Agency Bulletin 730 printed in 1973. The new publications update the guidelines for mathematics programs. They reflect significant changes in our society during the past decade and provide for the development of basic skills, including those needed now by students and those that will be essential in the future.

A focus of school mathematics on problem solving as the most basic skill has been brought about by the needs of the times. Advances in technology continually revolutionize our individual, social, and economic lives, causing some skills, once considered essential, to become obsolete.

Access to computers is growing, and the problem solving and instructional potential of computers and hand-held calculators has brought many changes into mathematics classrooms. The capability and widespread availability of these problem-solving tools cannot be ignored.

In recent years, there have been some distinct trends in elementary school mathematics programs. Reflecting the trends, certain topics in these guidelines have been modified to increase relevance to business and social life, thereby making number experiences meaningful and practical for students.

The revised program was developed by mathematics educators presently working with local and state instructional programs. These guidelines were reviewed by numerous teachers and administrators, and adjustments were made based on their recommendations.

This publication is to be used by school districts in designing or updating local curriculum and instructional programs, in accordance with Principle 6 (Rule .250) of ***Principles, Standards, and Procedures for the Accreditation of School Districts*** (TEA Publication AD0 824 01).

Raymon L. Bynum
Commissioner of Education

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Introduction

Program Goals

Mathematics permeates all sectors of life. Practical situations and economic needs are factors that determine the amount and kind of mathematics to be taught. The school must assume the responsibility of giving students the mathematical skills and information they will need as children and as adults.

In *An Agenda for Action: Recommendations for School Mathematics of the 1980s* (see Appendix A), the National Council of Teachers of Mathematics suggests ways for schools to address this responsibility. The curriculum recommendations include that:

- problem solving be the focus of school mathematics in the 1980s
- basic skills in mathematics be defined to encompass more than computational facility
- mathematics programs take full advantage of the power of calculators and computers at all grade levels
- more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population

Problem Solving

The most important goal of mathematics instruction is the development of the student's ability to solve problems. Emphasis should be placed on those problems that arise from real-life settings. The problem-solving process should require the student to use more than recall or previously established patterns of action to find a method for arriving at a conclusion. The skills of mathematics and the correct application of mathematical principles and methods are essential to this process.

The facility to apply previously learned mathematical principles, skills, and methods to real-life situations is important as well as is the achievement of prerequisites for continued formal study (and/or independent study) in mathematics, related fields, and other subjects using mathematics.

Basic Skills

The full scope of what is basic should contain at least the 10 basic skill areas identified by the National Council of Supervisors of Mathematics' "Position Paper on Basic Skills." These areas are:

- problem solving
- applying mathematics in everyday situations
- alertness to the reasonableness of results
- estimation and approximation
- appropriate computational skills
- geometry
- measurement
- reading, interpreting, and constructing tables, charts, and graphs
- using mathematics to predict
- computer literacy

Use of Calculators and Computers

It is recognized that a significant portion of instruction in the early grades must be devoted to the direct acquisition of number concepts and skills without the use of calculators. There are times, however, when the use of calculators is appropriate in instruction.

A calculator should be used when:

- it extends personal capability to do problem solving
- it speeds up a search for patterns
- it enables more examples to be considered
- it removes the anxiety of computation failure

- it permits attention to be directed to other problem aspects
- it provides motivation and confidence that a problem can be solved
- it offers access to mathematical ideas not available without it
- it eliminates tedious or routine computations
- it allows problems of greater intricacy to be attempted
- it contributes to further applications and explorations of related topics
- it saves time within the curriculum without significantly detracting from other objectives

Meeting Student Needs

Excellence in mathematics education is equally important for students of all needs and abilities. Arithmetic proficiency should not be used as a prerequisite to conceptual study and application of mathematical ideas. Such a prerequisite has condemned many low-achieving students to a succession of courses that seldom progress beyond drill in arithmetic skills.

An effective mathematics program should emphasize the development of meaning and understanding as a foundation for the manipulation of numbers. Many firsthand experiences and concrete materials should be used to provide the student a basis for understanding numbers and explaining mathematical processes.

Perhaps the one factor most essential to the success of the mathematics curriculum is an emphasis on understanding. It is no longer deemed sufficient for a teacher to be satisfied with competent computational performance by a child in spite of the obvious necessity for such skill. The need to find efficient, enjoyable, illuminating methods of instruction has led to the consensus that clear, penetrating understanding of certain essential mathematics should precede, but not supplant, computational skills. Educators agree students should traverse from the known to the unknown; from the particular to the general; from the concrete to the abstract. The commitment of mathematics educators is to create a school environment that helps each student to achieve mathematical literacy and to exercise, under proper guidance, the option to continue or to terminate the formal study of mathematics.

Fundamental Elements

Since all students do not attain the same levels of understanding and skill simultaneously, any program should possess three fundamental properties:

- **Flexibility**
  - to provide opportunities for students with minimum concepts to extend their basic knowledge without gaps
  - to provide ample stimulation and challenge for the gifted and talented to extend and enrich their learning experiences
  - to adjust to the rapid changes in a demanding and progressive world
- **Continuity**
  - to eliminate unnecessary repetition while strengthening existing weaknesses
  - to fill in basic concepts and skills
  - to extend the knowledge and skills of all students to the utmost by a uniformly progressive development
- **Correlation**
  - to integrate arithmetical, algebraic, and geometric concepts and applications
  - to apply mathematical ideas in other fields

However, the teacher of mathematics is the most important component of any mathematics program. The teacher must have knowledge of mathematics, utilize a variety of teaching styles and materials, and design appropriate strategies to facilitate the mathematics education of students. Moreover, the teacher of mathematics must exhibit positive attitudes toward mathematics, show respect for learners, exhibit enthusiasm for teaching, and have a realistic opinion of his or her own competencies.

Individualizing Instruction

An effective mathematics program should provide for student differences in achievement, maturity, learning rate, learning style, emotional and social behavior, physical conditions, and interests.

To provide for individual differences, mathematics teachers, administrators, and parents must support a personalized instructional program in mathematics. Such a program can take many forms; however, the existence and extent of individual differences must first be determined. It should be recognized that students at all levels of learning need to be offered variations of instruction. The method of subjecting students to the constant loneliness of working on instructional packets, day in and day out, could prove counterproductive. This method is not individualization but solitary confinement. It should also be recognized that groupings of students are valid for some topics of mathematics instruction while completely irrelevant for others.

The following techniques are successfully being practiced in many classrooms:

- Grouping students within the classroom according to their levels of mathematical development and presenting to each group subject matter that fits the group in scope and/or depth
- grouping students from several classrooms for large group presentations by teachers cooperating and utilizing each other's strengths
- grouping students by teams of two or more in which the leadership role is assumed by the students under the teacher's guidance
- making flexible and challenging assignments that are both real and practical to the student's life experiences
- providing a special setting for the study of mathematics where aid from experienced teachers of talented students is readily available
- providing a well-equipped mathematics laboratory that will supply appropriate alternate materials

Other suggested activities for differentiating instruction and caring for individual differences are:

- individual and group reports on topics investigated, making use of mathematics books available in the school library for the general student as well as the specialized student
- application of mathematics in solving school and community problems
- use of supplementary textbooks
- mathematics clubs
- mathematics exhibits and contests
- construction models and other visual aids
- activities that promote parent-student interaction

Using a well-devised plan of personalized instruction will help teachers of mathematics provide students an informational, inspirational, enjoyable, and worthwhile learning experience throughout their school careers.

Using the Program Guide

Principle 6c of *Principles, Standards and Procedures for the Accreditation of School Districts* states that:

> Program guides and curriculum frameworks issued by the Texas Education Agency are used by the district as references for designing local curriculum and supportive arrangements.

Since the program presented in this publication was developed for statewide use, specific placement and sequencing of objectives should be determined by identified needs in the local district.

Certain objectives have been identified as essential for all students. The Texas Assessment of Basic Skills (TABS) measures student achievement on these objectives. See Appendix B for correlation of the TABS objectives to this program.

The essential objectives do not represent a total mathematics program. Districts may choose to emphasize these objectives but should take advantage of the opportunities to develop these skills within the context of various topics. For example, multiplication skills are needed in the counting procedures used in probability.

This publication can be used as a resource for developing a district's mathematics program. The objectives are organized according to grade levels. The looseleaf pages can be separated and placed in a notebook according to grade level or topic. Space is provided at the right of each activity for identifying available resource material and instructional aids and for comments.

Instructional Aids

Instruction in mathematics can be made more effective through the proper use of suitable teaching aids. Mathematics is primarily concerned with ideas and abstractions, but an understanding of the ideas grows from concrete experiences. Teachers who keep abreast of innovative instructional materials and use them properly to enhance their instruction are most likely to be more effective in reaching a greater number of students.

Instruction in mathematical principles develops progressively from active, concrete experiences to increasingly complex mathematical symbolism. Active learning, student involvement, student participation, and relevance are educational concepts that receive the support of present-day educators. Concerned instructors are trying to teach students to relate mathematical ideas and symbols to real objects to provide a sense of realism. The use of mathematics resource centers, learning centers, or laboratory rooms are all examples of this approach to learning mathematics.

Teachers should use discrimination in choosing from the wealth of aids available for classroom use. The following list suggests instructional aids needed for implementing the program for Grades 6-8

- Overhead projector and transparencies
- Place-value charts or kits
- Number lines
- Hundreds chart
- Number games and puzzles
- Objects (toothpicks, counters, etc.) for manipulative work in multiplication, division, factoring
- Fraction models or kits
- Calculators
- Metric and customary measuring tools:
  - rulers
  - meter and yard sticks
  - spring and balance scales
  - thermometers
  - measuring spoons and cups

- Geometric models (two- and three-dimensional)
- Centimeter and inch squares and cubes
- Geometric puzzles (some cubes, tangrams, etc.)
- Art supplies (for making charts, graphs, games)
- Grids and maps
- Geoboards
- Compasses and protractors
- Resource materials:
  books
  charts
  pamphlets
  newspapers
  magazines

Evaluation

The first purpose of meaningful evaluation in school mathematics should be the improvement of learning programs, teaching, and materials. Educators must evaluate to have information for sound decisions, to be accountable to their public, and to know how well they are doing.

Evaluation is an important part of the instructional program in school mathematics. Unlike its traditional role, evaluation should be more than a terminal judgment of a student's work for grading purposes. Its role should be to determine how well the basis of the instructional program, the goals and objectives, are being and have been achieved by the student and to identify difficulties that may be encountered by the student in the learning processes.

Emphasis on individual achievement brings the need for a new focus on evaluation. A measure (that is independent of the performance of other students) is required to provide information on the degree of competence attained by an individual. Norm-referenced tests are useful for ordering students and making comparisons, but their ability to assess a student's achievement of specified curriculum objectives is limited. Criterion-referenced measurement can provide explicit information as to what students already know and where new learning needs to begin. This diagnostic aspect is valuable since the criteria that are used are the program's predetermined objectives. This type of diagnosis is also *vital* in determining what kind of mathematical learnings would ensure a student's continuous progress. In selecting or developing test items, care must be taken to see that they relate to the objectives of the mathematics program in use by the local school. An item of a test can measure the degree to which an individual has achieved an objective.

The recording procedure must accommodate student progress in relation to objectives. Such a procedure offers the teacher a means of viewing students in their individual progress toward established objectives for mathematics and toward their educational goals. Resulting reporting procedures should focus on student progress rather than comparison with a group or predetermined norm.

Evaluation is not limited to testing. It includes gathering data and interpreting the data. Testing is one source of data. There are many others. Today, many people use test scores as the sole index of the quality of mathematics programs or of the success of student achievement. Test scores alone should not be considered synonymous with achievement or program quality. A danger to the education of our youth is the increasing tendency on the part of the public to assume that the sole objective of schooling is a high test score. This assumption is made often without the critical knowledge of what is being tested or whether test items fit desired goals.

To perform the various aspects of the evaluation process, the teacher must use various techniques to aid in evaluation.

- Classroom observation and interaction are extremely useful because they provide immediate feedback for both the teacher and the student. These techniques are relatively easy to apply and can provide a great deal of information in a short period of time.

- Homework analysis allows for relatively immediate feedback. It permits a student to make a mistake without fear of a lower grade. This is especially important, for if students are to develop powers of inquiry and discovery, they must be allowed to work in nonthreatening situations where they can try different ideas.

- The evaluation of problem-solving performance will demand new approaches to measuring. Certainly present tests are not adequate. In particular, the measuring of the use of problem-solving processes will demand innovative techniques. Evaluation of programs with problem-solving goals must be sensitive to the nature of those goals.

Another important area of evaluation is that of student attitude. While attitude does not correlate with achievement, it is safe to say that a negative attitude may cause students to achieve at a lower level than the level at which they are capable of working. A student's attitude towards mathematics often may be observed directly by the teacher. This information then may be used by the teacher, if necessary, to help students change their attitude to a more positive one.

Attitude scales or open-ended questionnaires concerning how a student feels about mathematics may also be used. It must be remembered that the responses to the scales and questionnaires must be judged in light of what the teacher knows about the student from personal interaction so that the results may be judged appropriate or inappropriate.

A complete evaluation program must consider the effectiveness of the teacher as well as the effectiveness of the instructional program. Therefore, assessment in several areas should include:

- teacher effectiveness
- program effectiveness
- student achievement
- student attitude toward mathematics

To evaluate teacher and program effectiveness the teacher may ask questions such as:

- Did the teacher and program provide for individual differences?
- Did the teacher and program provide for each student's social development?
- Were the goals and the objectives of the instructional program appropriate for each student?
- Did the evaluation program provide a valid measure of the instructional program?

For a suggested procedure of program evaluation, see *Integral Elements of an Effective Mathematics Program*, TEA Publication CU1 832 05.

It is imperative that the goals of the mathematics program dictate the nature of the evaluations used to assess program effectiveness, student learning, teacher performance, and the quality of materials. Too often the reverse is true: either the tests dictate the programs or assumptions of the evaluation plan are inconsistent with the program's goals.

Overview: Program Matrix

Scope and Sequence

TOPIC	6	7	8
Number, Numeration, and Place Value	• History of numeration systems • Exponential notation • Rounding • Factoring (primes and composites) • Fractions, decimals, percents, ratio • Least common multiple • Greatest common factor • Integers	• History of number systems • Rounding • Scientific notation • Factoring (primes and composites) • Least common multiple • Greatest common factor • Fractions, decimals, ratio, percent • Integers	• Exponential notation • Ordering • Rounding • Significant digits • Scientific notation • Factoring • Rational numbers and properties • Ratio and percent

Operations and Properties	• Integer and decimal
	  • addition
	  • subtraction
	  • multiplication
	  • division
	• Fraction
	  • addition
	  • subtraction
	  • multiplication
		• Rational number
		  • addition
		  • subtraction
		  • multiplication
		  • division
		• Exponents
			• Rational number
			  • addition
			  • subtraction
			  • multiplication
			  • division
			• Absolute value

Problem Solving	• Estimation • Solving equations • Basic operations • Geometry • Measurement	• Estimation • Solving equations and inequalities • Basic operations • Geometry • Measurement • Denominate numbers • Proportion and percent	• Estimation • Order of operations • Writing and solving equations • Measurement • Geometry • Proportion and percent • Graphing • Probability and statistics • Personal finance
Measurement	• Metric and customary units • Perimeter and circumference • Area • Surface area • Volume • Conversion (within metric system) • Applications	• Metric and customary units • Conversion (within metric system) • Perimeter and circumference • Area • Surface area • Volume • Applications	• Metric and customary units • Conversion (within metric system) • Precision • Greatest possible error • Applications

Geometry	• Two- and three-dimensional figures
	• Congruence
	• Translations
	• Reflections
	• Rotations
	• Applications
		• Points, lines, rays, angles, planes
		• Properties and definitions of two- and three-dimensional figures
		• Similarity
		• Congruence
		• Symmetry
		• Geometric constructions
		• Applications
			• Right triangle geometry
			  • Pythagorean property
			  • similarity
			  • indirect measurement
			  • sine, cosine, and tangent ratios
			• Applications
			• Geometric constructions
			• Two- and three-dimensional figures
			  • area, surface area
			  • volume

Graphing, Relations, Functions	• Coordinate graphing
	• Construction of line graphs
	• Applications
		• Coordinate geometry
		  • similar figures
		  • reflection images
		  • linear equations
		• Applications
			• Coordinate geometry
			  • translations and reflections
			  • linear equations
			  • non-linear graphs
			• Applications

Probability and Statistics	• Gathering and organizing information • Interpretation and construction of charts and graphs • Sample spaces • Fractional probability • Averaging • Applications	• Frequency distributions • Mean, median, and mode • Interpretation and construction of charts and graphs • Simple event probability • Applications	• Counting methods • Factorials • Venn diagrams • Simple and compound events • Tables and graphs • Mean, median, mode • Predictions • Applications
Computer Literacy			• History of computers and calculators • Limitations and strengths

The Mathematics Program

Grade 6

Unit Title: NUMERATION AND PLACE VALUE

Objectives	Suggested Teaching Activities	Resources/Comments
1. Compare other numeration systems with the decimal numeration system.	Have students research other methods of numeration (Egyptian, Babylonian, Mayan, etc.)	
2. Read and write whole numbers to 10 digits.	Review place-value concepts using place-value chart and expanded notation. Have a student read excerpts from a world record book while other students record the numbers read.	
3. Name the place value and the period of each digit of a 10-digit numeral.	Ask students for examples of things that represent large numbers. Hand out a place-value chart. Write the width of our solar system on the board: 7,350,540,000 miles. Using the place-value chart, have students write this number in the correct places. Covering all the numbers on the left except the last three, decide which period it is, uncover next three, etc.	
4. Find a^b when a and b are whole numbers.	Identify the base number and the exponent. Begin with numbers such as 3^2, 2^3, 4^2, etc., and explain that in 3^2 the exponent 2 tells us to use 3 as a factor two times: $3 \times 3 = 9$	
5. Write in expanded notation a 10-digit number using powers of 10, and write a number in standard form from one in exponential notation.	Review the two earlier forms used to write a number in expanded notation. EXAMPLE: $4{,}376{,}241 = 4{,}000{,}000 + 300{,}000 + 70{,}000 + 6{,}000 + 200 + 40 + 1$ and $(4 \times 1{,}000{,}000) + (3 \times 100{,}000) + (7 \times 10{,}000) + (6 \times 1{,}000) + (2 \times 100) + (4 \times 10) + (1 \times 1)$ Then ask students to write each multiple of 10 as 10 with an exponent. Substitute the exponent form in the second method, and the result is a number in exponential notation using powers of 10.	
6. Use the symbols >, <, or = to compare two whole number expressions.	After discussing the terms greater than, less than, and equal to, point out that if a number is to the left of another on the standard number line, then it is < . A number to the right of another number is > Point out that the open end of the > mark is always open to the larger number. Ask class to discuss reasons why it is written in this way.	

7. Round a numeral with four (*** to 10) digits to the nearest 10, 100, 1,000 (*** 10,000 or 1,000,000).

Let the class draw number lines for 0-100 counting by tens, 0-1,000 counting by hundreds, and 0-10,000 counting by thousands.

Approximate the location of numbers: e.g., 3, 38, 58, 401, and 799 on all number lines and show on all three (counting by tens, hundreds, and thousands) whether each number will round to the nearest 10, 100, or 1,000. (Some numbers cannot be shown on the first two number lines.)

Discuss numbers ending with 5, 50, 500. Point out that these numbers are usually rounded up to the next 10, 100, or 1,000.

8. Read and write decimal numerals through ten-thousandths.

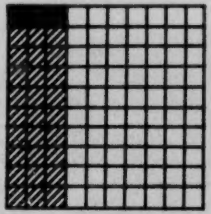
Emphasize that a decimal is a symbol for a fractional number. Point out that .2, .12, .125 are decimal fractions, usually called decimals. The students should be aware that each position has a place value and should name those places through ten-thousandths.

Next show the process of reading correctly with *and* being the decimal and the last word naming the place value of the last digit in the number.

Give students several examples written both with numerals and with words. Call orally several for students to record, then check.

9. Order an appropriate set of decimal numbers from least to greatest.

When the student has a meaningful knowledge of the place value, the student can order with limited difficulty. The number line can be used to show decimals with some numbers to the right of the decimal point. Show students equality of such numbers as .3, .30, .300. Use the hundreds chart:



$.3 = \frac{3}{10}$ $.30 = \frac{30}{100}$ 30 out of 100 is the same as 3 out of every 10 so $\frac{30}{100} = \frac{3}{10}$

Then expand the number line to compare, for example, that 7 is greater than .6. Write a comparison in the place-value chart. Read as "six tenths" and "seven (ones) and no tenths."

ones	tenths
	.6
7	.0

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NUMERATION AND PLACE VALUE, continued

Objectives	Suggested Teaching Activities	Resources/Comments
10. Round a decimal number to tenths, hundredths, or thousandths.	Use the number line, as in rounding to the nearest 10, 100, etc. Write the decimal .16725 on the board, and have the class tell what each digit's value is. Also, do the reverse procedure and ask what digit is in the hundredths' place, tenths' place, etc. Now go through the rules of rounding. If the digit to the right of the place being rounded to is 0 through 4, the digit in the place being rounded to remains the same. If it is 5 through 9, the digit in the place being rounded to is rounded up. Give several examples rounded to different place values.	

Unit Title: ADDITION AND SUBTRACTION OF WHOLE NUMBERS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Use games and activities to reinforce the addition and subtraction facts.	Provide a variety of ways for practice in the basic addition and subtraction facts. One example: Have students make a grid for a multidigit, multinumber addition problem as below: $\begin{array}{r} \square\square\square \\ \square\square \\ \underline{+\,\square\square\square} \end{array}$ Students may fill in the boxes with numbers randomly drawn or called out and then perform the addition. The student with the greatest sum wins.	
2. Identify the commutative (order), associative (grouping), and identity (zero) properties of addition.	Use very simple illustrations to review these properties. Play a game in which a student makes up an example and another student tells which property it is. This activity may be done in a team or on an individual basis.	
3. Find the sum when given a problem with up to 5 four-digit numbers.	Students may be led through the steps in arriving at the shortcut method of addition. First, the problems may be solved by expanded notation. For example: $\begin{array}{r} 37 = 30 + 7 \\ \underline{16 = 10 + 6} \\ 40 + 13 = 53 \end{array}$ In the second step, students may record the addition: $\begin{array}{r} 37 \\ \underline{+\,16} \\ 13 \\ \underline{+\,40} \\ 53 \end{array}$ Finally, students arrive at the shortcut where the 1 ten in 13 is regrouped with the other tens: $\begin{array}{r} 1 \\ 37 \\ \underline{+\,16} \\ 53 \end{array}$	

ADDITION AND SUBTRACTION OF WHOLE NUMBERS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
4. Write the difference, regrouping when necessary, of two numbers having up to five digits.	Review subtraction as the process of undoing addition. Discuss using words such as "the opposite of addition" or "the inverse of addition." Have students write a mathematical paragraph about the facts: $7 + 8 = 15$ $8 + 7 = 15$ $15 - 7 = 8$ $15 - 8 = 7$ Have students illustrate a subtraction problem such as 71 – 54 using markers on a place-value chart. Draw attention to the needed exchange of 1 ten marker for 10 one markers before the subtraction can be done. Discuss how this manipulation would be recorded.	
5. Estimate sums and differences.	Review rounding procedures and use estimation to predict results found with the calculator.	
6. Use inverse operations to solve equations.	Discuss the opposite of putting on a hat, taking off a coat, getting on a bus, etc. Another word for opposite would be inverse. Ask what the inverse of addition would be; the inverse of subtraction? Show the following examples: $N + 9 = 17$ $X - 10 = 7$ $N = 17 - 9$ $X = 7 + 10$ $N = 8$ $X = 17$	
7. Use various strategies for problem solving.	Review the steps in problem solving: a. Understand exactly what information is given and what is to be found. b. Make a list of the given and derived information, or use a chart, diagram, or other visual model. c. Decide what information is relevant. d. Devise a plan. e. Implement the plan to arrive at tentative conclusions. f. Test the conclusions. g. Revise the plan, if necessary, and proceed as before. Challenge the class to make up problems for each other and to critique each others' solutions according to the above criteria.	

Unit Title: MULTIPLICATION AND DIVISION OF WHOLE NUMBERS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Use games and activities to reinforce the multiplication facts.	Provide a variety of ways for practicing the basic multiplication facts, including games, contests, and problem solving.	
2. Identify an equation as an application of the commutative (order), associative (grouping), or identity property of multiplication.	**Commutative Property:** Have class show with manipulatives (or on graph paper) simple multiplication facts, such as 3 x 5. Then have them show with manipulatives (or on graph paper) the related fact (5 x 3). Have students count the number of manipulatives (or squares on the graph paper) to show that the products are equal. **Associative Property:** Have students choose three numbers between 1 and 10. Multiply the first two and make an array showing this fact. Draw this array as many times as is indicated by the third number. Next have students group the third number with one of the first numbers. Make an array with these two numbers. Draw this new array as many times as is indicated by the unused number. **Identity Element:** Ask class what the products are for 786 x 1? 32 x 1? 1,163,211 x 1? If y represents any whole number, what is y x 1? What is any whole number times 1? (Itself.) What is your identity? Mary? Jeff? Sue? The number 1 is the *identity element* for multiplication.	
3. Write the product for a multiplication problem with a factor of zero or write zero as a missing factor when the product is zero.	Question the students: "What happens when you multiply 4 and 0?" Show on the number line the quantity 4 zero times. Show the quantity 0 four times.	
4. Write the product for a multiplication problem in which one factor is a one-digit multiple of some power of 10 (90, 300, etc.).	Have students work some problems with one factor as a multiple of a power of 10: $\begin{array}{r}3{,}000\\ \underline{\times 5}\end{array} \qquad \begin{array}{r}80\\ \underline{\times 90}\end{array}$ Let them review the basic facts of the problems (3 x 5, 8 x 9) and use these basic facts to discover the pattern in this type of multiplication problem.	

Grade 6

MULTIPLICATION AND DIVISION OF WHOLE NUMBERS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
5. Find the product for a multiplication problem with multidigit factors.	For a problem such as 237 x 58, have students name the hundreds, tens, and ones in the first factor and solve for the number of hundreds, tens, and ones when multiplied by the 8 and 50 in the second factor. Have them record this procedure in expanded notation: $\begin{array}{r} 237 = 200 + 30 + 7 \\ \underline{x58} = \underline{\qquad\qquad 50 + 8} \\ 1{,}600 + 240 + 56 \\ \underline{10{,}000 + 1{,}500 + 350 \qquad} \\ 10{,}000 + 3{,}100 + 590 + 56 = 13{,}746 \end{array}$ Use the concepts from the expanded notation form to develop the shortcut method for recording this type of multiplication problem. Beat the Clock: Draw a circle on the blackboard with four numbers having two or three digits around the edge, and one number having three or four digits in the middle. Set a time limit of 8 to 12 minutes and challenge the students to multiply each outer number by the number in the center before the time is up.	
6. Relate multiplication and division by finding missing factors.	Given a product and a factor (basic facts) the student can arrange beans or beads to form an array with one side being the dimension of one factor. The dimension on the other side will show the missing factor.	
7. Show that division by zero is meaningless.	Give several division problems. Then give $4 \div 0$ or $0\overline{)4}$ and ask how many groups of zero can one make out of four things? Hand out toothpicks for the amount of the dividend and let students try to show how many groups they can get. Since division is related to multiplication, $0\overline{)4}$ means 0 x _______ = 4. There is no whole number that satisfies this equation; therefore, division by zero is undefined, or impossible.	

8. Divide a multidigit dividend by a one-digit divisor.

Review basic division facts and their inverse multiplication facts.

Review the concept of division with concrete objects.

Use a pocket chart to demonstrate the problem 142 ÷ 2 = n. Beginning with the Hundreds pocket, ask the student to try to divide the number of strips in each pocket into two groups, each containing the same number. In the H pocket, there is only one strip, so division cannot be made. Have the student remove the strip in the H pocket and replace it with 10 strips in the Tens pocket, making 14 strips in the T pocket. Have a student explain this action. Now have the student divide the 14 tens into two groups, each containing 7 tens; then divide the two strips in the Units pocket into two groups of one each, placing one with each group of 7 tens. By counting the 7 tens and 1 unit in each group to obtain 71, have the student show that 142 ÷ 2 = 71. Then write the problem on the chalkboard.

Sometimes students need a starting place to help them perform the operation of division in the abstract form. The following step-by-step approach using the repeated subtraction process may be used with students.

With 80 objects divided into sets of four, how many sets can I get? Can I get 100 sets? THINK: 100 x 4 = 400, so not 100 sets.
Can I get 10 sets? THINK: 10 x 4 = 40, so yes, at least 10 sets.

$$\begin{array}{r l} 80 & \\ \underline{-\,40} & \text{10 sets of 4} \\ 40 & \\ \underline{-\,40} & \underline{\text{10 sets of 4}} \\ 0 & \text{20 sets of 4 in 80} \end{array}$$

As the numbers increase, students are led to think in terms of 100 sets or multiples of 100 sets at a time. For example:

650 objects divided into sets of 5

$$\begin{array}{r l} 5\,\overline{)650} & \\ \underline{-\,500} & 100 \times 5 \\ 150 & \\ \underline{-\,150} & \underline{\;30 \times 5\;} \\ 0 & \text{130 sets of 5} \end{array}$$

MULTIPLICATION AND DIVISION OF WHOLE NUMBERS, continued

Objectives	Suggested Teaching Activities	Resources/Comments

The process of dividing is one that is difficult for many students. The method of division by repeated subtraction is one that gives students the opportunity to understand division by removing and subtracting equal sets. This method may take either one of two forms:

$$\begin{array}{rrr}
(1) & 4\,\overline{)676} & \\
 & \underline{-400} & \textcircled{100} \\
 & 276 & \\
 & \underline{-240} & \textcircled{60} \\
 & 36 & \\
 & \underline{-36} & \underline{\textcircled{9}} \\
 & 0 & \textcircled{169}
\end{array}
\qquad\qquad
\begin{array}{rr}
(2) & \underline{169} \\
 & 9 \\
 & 60 \\
 & \underline{100} \\
 & 4\,)676 \\
 & \underline{-400} \\
 & 276 \\
 & \underline{-240} \\
 & 36 \\
 & \underline{-36} \\
 & 0
\end{array}$$

The second form may provide an easier transition to the shortcut method when it is presented. Either form could be used throughout the presentation of the division algorithm.

Give practice using the same approach with problems that have remainders. For example:

$$\begin{array}{rr}
4\,\overline{)46} & \\
\underline{-40} & 10 \times 4 \\
6 & \\
\underline{-\ 4} & \underline{1 \times 4} \\
2 & 11\ \text{R2}
\end{array}$$

9. Divide numbers with the dividend and divisor in multiples of 10 or 100.

Put 30 x 20 = 600 on the board. Ask, "What related division equations can we write from this equation?" (600 ÷ 30 = 20, and 600 ÷ 20 = 30.) Give the students several problems, such as _______ x 10 = 200 or 20 x _______ = 200. Ask them to supply the missing numerals and label the numerals as to whether they are factors or products. Then write the related division equations from these same sentences. Again ask students to label the numerals as to whether they are factors or quotients. They should see that when they have found the missing factors in the first equations, they have found the quotients in the second equations.

10. Find the quotient of a four-digit dividend and a two-digit divisor without and with remainders.

Two-digit divisors are handled with estimation. The divisor is thought of as a multiple of 10: 23 as 20, 44 as 40, 212 as 200, etc. If the divisor is a number such as 47 and is rounded to 50, the guess for the quotient may be out of line. Therefore, 47 may be thought of as 40, 36 as 30, etc.

30
29) 213

THINK

Can I get 10 sets of 29?
10 x 30 = 300. No.

THINK

7 x 30 = 210
7 x 29 = 203

− 203 7 x 29
10

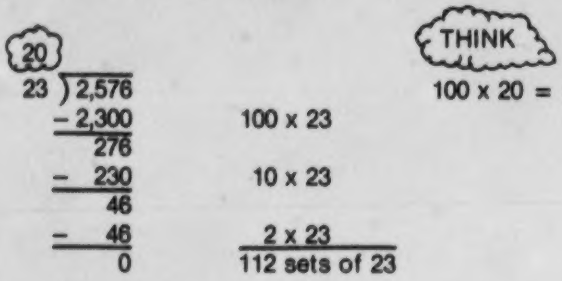
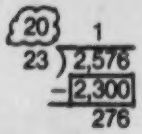
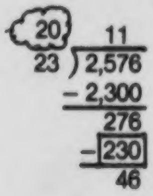
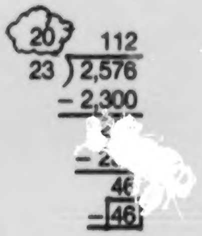
7 sets of 29 with 10 objects remaining.

Use the same procedure with larger dividends.

The shortcut in division is presented only after the students are proficient with two-digit divisors.

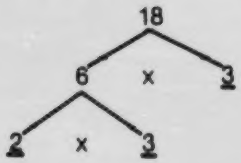
Grade 6

MULTIPLICATION AND DIVISION OF WHOLE NUMBERS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
	As the shortcut is presented, it is explained step-by-step, with reference to the procedure used previously. Place value is emphasized.     Because students will use the division algorithm in future computations, importance should be placed on understanding of the division rather than on use of the shortcut method in recording the division. Use a flow chart method to show the process of division. Have students complete each step on the chart. Use some problems for which all work is done except the quotient, all is done except products, and all is done except differences; have students fill in the missing numbers.	

	For some problems completely worked with mistakes in the work, have students find errors. The students may want to get in teams of two or four and have a contest to see which team can correct the problems first.
11. Estimate products and quotients.	Review rounding procedures and use estimation to predict results found with the calculator.
12. Use inverse operations to solve equations.	Discuss the meaning of *inverse*. What is the inverse of multiplication? Of division? Use the following examples: N x 6 = 42 X ÷ 9 = 13 N = 42 ÷ 6 X = 13 x 9 N = 7 X = 117
13. Use various strategies for problem solving.	Review the steps in problem solving: a. Understand exactly what information is given and what is to be found. b. Make a list of the given and derived information, or use a chart, diagram, or other visual model. c. Decide what information is relevant. d. Devise a plan. e. Implement the plan to arrive at tentative conclusions. f. Test the conclusions. g. Revise the plan, if necessary, and proceed as before. Challenge the class to make up problems for each other and to critique each others' solutions according to the above criteria.

Unit Title: FACTORING

Objectives	Suggested Teaching Activities	Resources/Comments
1. Determine if a number less than 100 is prime or composite.	Find the factors of several numbers. Ask students to group all of those with similar characteristics (one group of those with two factors, one group of those with more factors than two). Define prime and composite. Divide into teams. Team 1 thinks of a number and says "prime" or "composite." Team 2 makes a guess. Team 1 replies "higher" or "lower." Keep a record of time. The team with the least amount of time for guessing the number wins. Have each student make a Prime Number Chart.	
2. Write a number less than 100 as a product of prime factors (***using exponents).	Review prime numbers. Ask students to find the set of factors of 18, using the factor tree. Have students underline the prime factors so their tree can't grow from that branch anymore.  $18 = 2 \times 3^2$	

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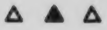
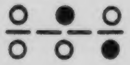
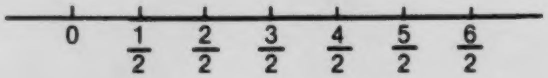
Unit Title: FRACTIONS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Identify and define the numerator and denominator of a common fraction.	After working with number lines, drawings, and a physical set of objects, note for students that the *numerator* in each fraction names how many parts are used and the *denominator* tells the number of parts in a whole. NOTE: Students should be encouraged to state this generalization in their own words even before it is presented by the teacher. Point out the location of the numerator and the denominator, and give several other fractions so students can point them out and explain their meanings.	
2. Illustrate a fraction as a division problem and a division problem as a fraction.	Write a fraction on the board, such as $\frac{2}{3}$, and explain that it can be read: two divided by three. Show that it can also be written as $3\overline{)2}$. Use manipulatives to illustrate a fraction as a division problem. To divide two candy bars evenly among three people, $3\overline{)2}$, the candy bars may be cut into thirds and distributed with each person receiving two of the thirds pieces, or $\frac{2}{3}$ of a candy bar.	
3. Identify equivalent fractional parts using an object or set of objects.	Review the factoring of a number into its prime factors. Draw several pictures of circular regions on the board and shade them to show why sentences like the following are true: $\frac{1}{2} = \frac{3}{6}$ $\frac{2}{3} = \frac{8}{12}$ $\frac{1}{4} = \frac{2}{8}$ Use a number line marked off from 0 to 3 with halves, thirds, fourths, and sixths marked off. Have students give various names for such fractions.	

FRACTIONS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
	Illustrate equivalent fractions with sets of objects:  $\frac{1}{3}$ of the set is black  $\frac{2}{6}$ of the set is black $\frac{2}{6} = \frac{1}{3}$ because in $\frac{2}{6}$, one out of every three $(\frac{1}{3})$ is black. Paper folding provides individual discovery of the meaning of fractions. Circles divided into equivalent parts fit into a cardboard circle. This aid helps students visualize the meaning of fractions. The number line helps show position. Both may be used to show equivalency.	
4. Use a number line, with appropriate points marked off, to write fractions to name the points indicated.	Display on the chalkboard a number line marked from 0 to 3, with divisions for halves and fourths. Draw an arrow to a particular point and work with students to name this point. Review terms *numerator* and *denominator* and how each is derived from the number line. Also have students observe that many points will have more than one name. Give the students several points to label independently, then check.  0 $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$	
5. Compare two fractions with like denominators using >, <, =.	Using physical representations of the fractions, the students can compare the models as to size to determine which is greater. Have students plot the fractions on the number line to understand what is meant by *larger* or *smaller*.	
6. Order a set of fractions with like denominators from least to greatest.	Write a set of fractions on the board such as $\frac{5}{6}$, $\frac{2}{6}$, $\frac{3}{6}$. Ask students to order mentally these fractions from smallest to largest. Write their guesses on the board. Now say, "There is a way to test these." Ask for ideas. If there are none, suggest using the number line.	

7. Determine the least common multiple of two numbers, each less than 100, to find the least common denominator.	Have students factor the numbers into their prime factors. Then look at each factor separately. For example, for 24 and 36: How many times does 2 appear as a factor in 24? in 36? Since it appears three times in 24, the least common multiple must have three factors of 2 also. (Three factors of 2 will include the two factors of 2 needed for 36.) Similarly, since 36 has two factors of 3, the LCM will need two factors of 3. (These two factors of 3 will cover the one factor of 3 in 24.) Therefore, LCM = $2 \times 2 \times 2 \times 3 \times 3 = 72$. This activity prepares students for finding the least common denominator when adding fractions with unlike denominators.
8. Compare two fractions with unlike denominators using $<$, $>$, $=$.	Have students plot the fractions on the number line to see their relative values. Then have them find the common denominator and use what they know about equivalent fractions to rewrite the two numbers as like fractions. $\frac{1}{4} = \frac{3}{12}$ $\quad \frac{1}{3} = \frac{4}{12}$ $\quad \frac{1}{4} < \frac{1}{3}$ because $\frac{3}{12} < \frac{4}{12}$
9. Order a set of fractions with unlike denominators from least to greatest.	Write a set of fractions on the board such as $\frac{5}{6}$, $\frac{3}{2}$, $\frac{2}{3}$. Ask students to order mentally these fractions from smallest to largest. Write their guesses on the board. Ask for ideas on how to check the guesses. If the students do not suggest any ways, mention using a number line or finding equivalent fractions with common denominators.
10. Determine the greatest common factor of a number less than 100 to prepare for reducing fractions to lowest terms.	Factor the given numbers into prime factors. Discuss which factors should be used to generate a number that will divide into both the given numbers evenly (a common factor). Students should arrive at the conclusion that only factors appearing in both given numbers can be used to generate a common factor. From this conclusion, lead students to see that the greatest common factor would be generated by using every factor common to both given numbers. $12 = 2 \times 2 \times 3$ $18 = 2 \times 3 \times 3$ $\quad$ GCF = $2 \times 3 = 6$
11. Rename a fraction in lowest terms.	Review the identity element of one. Have students factor both the numerator and denominator to find names for one in order to simplify the fractions. $\frac{12}{18} = \frac{2 \times 3 \times 2}{2 \times 3 \times 3} = 1 \times 1 \times \frac{2}{3} = \frac{2}{3}$ Point out that finding the names for one is the same as finding the greatest common factor of the numerator and denominator. Do examples using the GCF in renaming the fraction.

Unit Title: OPERATIONS WITH FRACTIONS

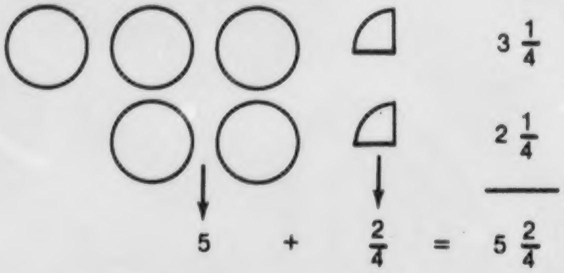
Objectives	Suggested Teaching Activities	Resources/Comments
1. Add and subtract fractions with like denominators.	Draw a circle on the board and divide it into fourths. Ask students to identify the parts. Then give a simple problem such as $\frac{1}{4} + \frac{1}{4}$. Show the addition by joining two of the one-fourth parts in the picture. Ask students to identify the sum. If the students say $\frac{2}{8}$, show that $\frac{2}{8}$ is the equivalent of $\frac{1}{4}$ making the answer of $\frac{2}{8}$ an unreasonable one. Have students do several simple problems using manipulatives or pictures before requiring them to write the problems. (Do not require students to reduce improper fractions.) Students may use the same activities to develop the concept of subtracting fractions with like denominators.	
2. Name the least common denominator of two unlike fractions.	Review the prime number method to find common denominators (least common multiple). Suggest that students first look for the common denominator as being one of the denominators (in $\frac{1}{4} + \frac{1}{8}$, eight is the common denominator). But sometimes it will not be found in either of the fractions. Then the students must use what they know about least common multiples and equivalent fractions.	
3. Add and subtract fractions with unlike denominators.	After students have had multiple experiences with addition of like fractions, present two fractions with unlike denominators. Use models—rectangles, circles, or equal pieces (i.e., cut one orange in halves, one in fourths). Have students give the total if one of the halves is put with three of the fourths. Help them to see that the half must be cut in two—making fourth pieces—to be able to express the answer. (Use the other half of the orange and cut it to show that each part is really one-fourth of the orange.)	
4. Change a mixed number into an improper fraction and an improper fraction into a mixed number.	Review fractions that name the number one.	

Draw two circular regions divided into eighths on the board. Shade one entire region and one-eighth of the other. Ask the students how many eighths are shaded? (nine-eighths) How many whole regions? (one) What fractional part of the second region? (one-eighth) Write $\frac{9}{8}$ and $1\frac{1}{8}$ on the board. Have students observe that the numbers are equal and lead them to develop a method for changing an improper fraction as a division problem.

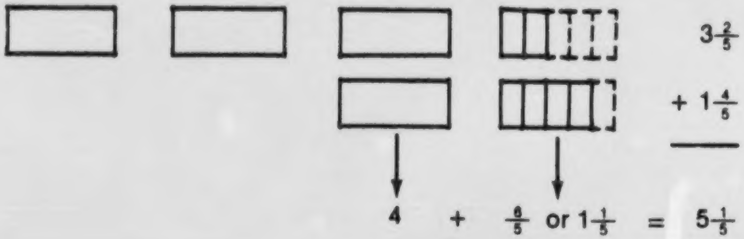
Review inverse operations and apply the inverse of the method to change a mixed number to an improper fraction.

5. Add mixed numbers.

Using models or pictures, demonstrate addition problems such as $3\frac{1}{4} + 2\frac{1}{4}$. Because $3\frac{1}{4} = 3 + \frac{1}{4}$ and $2\frac{1}{4} = 2 + \frac{1}{4}$, the order property of addition allows addition of the fractional parts first, then of the whole parts, and then of the whole and fractional parts.

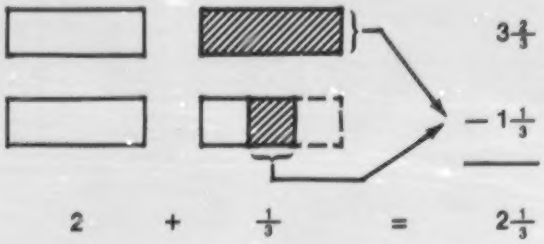
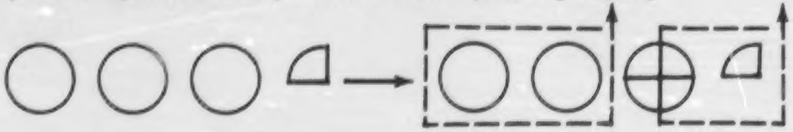


When students are comfortable with these manipulations and can record them, they are ready to move to problems that involve regrouping.



Students should be led to see that in problems with unlike denominators in the mixed numerals, the addition process is the same after a common denominator has been found for the fractional parts.

OPERATIONS WITH FRACTIONS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
6. Subtract mixed numbers.	Using models or pictures, demonstrate subtraction (without regrouping) of mixed numbers.  $3\frac{2}{3}$ $-1\frac{1}{3}$ $2 + \frac{1}{3} = 2\frac{1}{3}$ When students are comfortable with these manipulations and can record them, they are ready to move to problems that require regrouping.  $3 + \frac{1}{4} = 2 + \frac{5}{4}$ $-(2 + \frac{3}{4})$ $0 + \frac{2}{4} = \frac{2}{4}$ Students should work many problems with pictures or models before relying totally on symbols. For problems involving unlike fractions, students should be led to see that, after obtaining a common denominator, the procedure is the same as for like fractions.	

7. Name the product when one factor is a whole number and the other a fraction.

Use models to introduce multiplication of a fraction by a whole number (repeated addition).

$$5 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{5}{4}$$

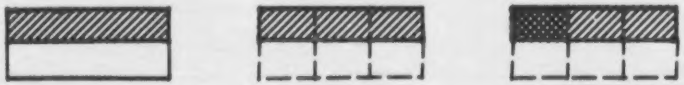
Illustrate work on a number line.

Now ask the students what would be an easy way to multiply 5 × $\frac{1}{4}$ using only those numbers. State that whole numbers can be made into fractions by placing them over one. Have the students test their easy method with several examples.

8. Find the product of two fractions.

Introduce the method of dividing rectangular regions into congruent parts to show multiplication. Example: $\frac{1}{3} \times \frac{1}{2}$.

1. Divide into rows according to second denominator (2).
2. Shade rows indicated by numerator (1) of second fraction. (Steps 1 and 2 represent a group of one-half.)
3. Divide region into columns according to first denominator (3).
4. Crosshatch the number of columns indicated by the numerator (1) of the first factor. (This represents $\frac{1}{3}$ of a group of $\frac{1}{2}$.)



$\frac{1}{2}$ finding thirds of $\frac{1}{2}$ shading $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$

Have students to determine area shaded and crosshatched($\frac{1}{6}$).Show several examples and have students work them until they begin to see the pattern in multiplication of fractions (the product of the numerators over the product of the denominators).

9. Find the product for a multiplication problem when at least one number is a mixed number.

Write an example such as $\frac{3}{4} \times 1\frac{1}{3}$ on the board. At this time, the student should observe it would be helpful to change the mixed numeral to an improper fraction before multiplying. Give students several examples in which to use this method.

Grade 6

Unit Title: DECIMALS, PERCENTS AND RATIOS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Write the decimal equivalent of a number in fractional form (limit to fractions that can be expressed as tenths or hundredths).	Use a hundreds chart on which students can indicate fractional parts. $\frac{1}{10} = .1$ or $.10$ $\frac{1}{5}$ of the board shaded is $\frac{20}{100}$ so $\frac{1}{5} = .20$ or $.2$	
2. Write a fraction as an equivalent fraction with a denominator of 100 and express it as a percent.	Place a simple fraction, such as $\frac{1}{2}$, on the board. Discuss the term *percent*. Make sure that the class knows that it is a certain number out of one hundred, or hundredths. Tell them that to find the percent that is equal to $\frac{1}{2}$, we must find its equivalent in hundreds by using the equation $\frac{1}{2} = \frac{N}{100}$. Have the students also write the answer in percent notation: $\frac{50}{100} = 50\%$. Concepts can be reinforced here, again, with a hundreds chart.	
3. Write equivalent fractions as decimals, fractions, and percents.	Define percent—another name for *hundredths*. The symbol 16% means sixteen hundredths. Emphasize that % means $\frac{1}{100}$ and that $16\% = \frac{16}{100}$. Ask in what other way $\frac{16}{100}$ could be written (.16).	
4. Read, write, and interpret ratios.	Divide class into three sets (i.e., brunettes, blondes, redheads). If 18 are brunettes, 5 are blondes, and 2 are redheads, show that brunettes are to blondes as 18 is to 5, and that this fact can be symbolized: $\frac{\text{brunettes}}{\text{blondes}} = \frac{18}{5}$, 18 to 5, or 18:5 All these symbols should be read 18 to 5. Other comparisons can be made with redheads to blondes, etc. or with other collected data.	

Unit Title: OPERATIONS WITH DECIMALS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Find the sum of decimal numbers through thousandths.	Review the place-value concepts with decimals. Use a place-value chart. Stress the importance of keeping the decimal points in line so that like terms can be added. Give the class several money problems to begin with. Emphasize that putting zeros to the right of the last digit does not change its value because none of the digits change place-value position. Give several examples such as 3.9 + 2.165.	
2. Find the difference of two decimal numbers through thousandths.	Stress the importance of keeping the decimal points in line so that like terms can be subtracted. To begin, give the class several money problems. Emphasize that putting zeros to the right of the last digit does not change its value. Give several examples, including problems that require filling in zeros for regrouping. A good application would be using a calculator and noticing the decimal in the answer.	
3. Find the product of a decimal factor and a whole number factor.	Write on the board a simple multiplication problem, such as 2 x .3. Rewrite the problem using an equivalent fraction and discuss the product as a fraction and as an equivalent decimal. $2 \times .3 = 2 \times \frac{3}{10} = \frac{6}{10} = .6$ $\quad$ $4 \times .12 = \frac{4}{1} \times \frac{12}{100} = \frac{48}{100} = .48$ After several examples, lead students to see that the first step in multiplying decimals is to multiply as with whole numbers. The second step is to determine the *denominator* of the decimal. $\begin{array}{r} 2 \\ \underline{\text{x}.3} \\ .6 \end{array} \qquad \begin{array}{r} .12 \\ \underline{\text{x}4} \\ .48 \end{array}$ The multiplication also can be illustrated with repeated addition: 2 x .3 is .3 + .3 = .6	

OPERATIONS WITH DECIMALS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
4. Find the product of two decimal factors.	Extend the previous activity to include two decimal factors. EXAMPLE: $.4 \times .3 = \frac{4}{10} \times \frac{3}{10} = \frac{12}{100} = .12$ $\begin{array}{rl} .4 & \text{1 decimal place} \\ \underline{\times .3} & \text{1 decimal place} \\ .12 & \text{2 decimal places} \end{array}$ After several examples using fraction representations, lead students to see that the first step in this type of multiplication is to multiply as with whole numbers. Then the decimal point in the product is determined by the number of decimal places in the factors. Give examples to point out that this general rule also holds in multiplying a decimal and a whole number. Reinforce the concept using problems involving *mixed* numerals, such as 4.21 x .5.	
5. Divide a decimal by a whole number.	The student previously has seen division involving all whole numbers and has a recent knowledge of placement of the decimal point in products. These two principles can be combined to help the student gain the correct knowledge of the decimal place for division. When the divisor is a whole number, the student only has to see that the quotient will involve as many decimal places as the dividend.	
6. Find the quotient when given a divisor showing tenths or hundredths and a dividend through thousandths. (Divisor should be no more than two digits.)	Write the problem $.8\overline{)7.2}$ on the board. Rewrite the division problem in fractional form: $\frac{7.2}{.8}$. Ask the students what can be done to turn this problem into a more familiar one. If someone says change .8 to 8, explain that this change can be made by multiplying by 10. If it is done to the divisor, it must also be done to the dividend to make an equivalent division statement (equivalent fraction). Although the concept may not be fully understood without more experience in division and fractions, the student should catch on readily about the moving of the decimal as many places to the right as may be necessary to make a whole number divisor and the accompanying move in the dividend. Show several examples using divisors that have different place values and be sure to include a type where additional zeros must be put on the dividend.	

7. Estimate sums, differences, products, and quotients.	Review rounding procedures and use estimation to predict results found with the calculator.
8. Use various strategies for problem solving.	Review the steps in problem solving: a. Understand exactly what information is given and what is to be found. b. Make a list of the given and derived information, or use a chart, diagram, or other visual model. c. Decide what information is relevant. d. Devise a plan. e. Implement the plan to arrive at tentative conclusions. f. Test the conclusions. g. Revise the plan, if necessary, and proceed as before. Challenge the class to make up problems for each other and to critique each others' solutions according to the above criteria.

Grade 6

Unit Title: MEASUREMENT

Objectives	Suggested Teaching Activities	Resources/Comments
1. Measure and draw line segments in centimeters, millimeters, inches, and fractions of inches.	Review the concept of measurement. Discuss the idea that a measure is a number and that measurement is most conveniently expressed as a number and a unit of measure. Students can measure strips of paper, pencils, erasers, etc. in meters and centimeters and can use meter sticks to measure the blackboard, doors, windows, etc. Students can measure strips of paper, pencils, erasers, etc. in feet and inches and can use yardsticks to measure the blackboard, doors, windows, etc. Have students do some mental measurement. (Estimate the dimensions of the room in feet, inches, and yards. A desk, a book.) Why are some unit measures more appropriate for the task? Ask students to guess lengths of objects in an envelope, then have them measure to test their guesses. Some objects should be measured in inches, some in centimeters or both. Record individual lengths. Prepare a duplication master with line segments. Ask students to measure each in millimeters and centimeters. Have them give answers as a set of equal measures: 3.2 cm = 32 mm. Have students choose the most sensible measure. On a transparency, prepare a list of objects and give three measurements. Student records choice. (Only one is the actual size or measurement.) Divide the room into six groups. Have students measure items in a box, exchange boxes with another group, and compare measurements. Ask students to look up the six metric prefixes in a dictionary and find the etymologies. Have them find other words that use these prefixes. Display the findings.	

2. Find the perimeter of a polygon by measuring or using given measures. (Use both metric and customary units.)

Draw a triangle, a quadrilateral, and a pentagon on the chalkboard (or use a transparency). Help students to classify the figures, leading them to discover that they are all polygons. After discussing the display, have students write a definition for the term perimeter. Defined as "the *distance* around a polygon," it is measured in linear units. Caution students to note that all measurements are approximations.

Have objects in the various shapes. Give different sets to groups of students. Have them measure and record the perimeter of each. Use inches and centimeters.

Use an almanac or encyclopedia to find the dimensions of such things as the Great Pyramid of Egypt, the Washington Monument, the Pentagon, the San Jacinto Monument. Have students find the perimeter of sides or bases. Use for a bulletin board display.

Have students find an irregular-shaped object or picture of one, make a scale of it, and display the object with dimensions and perimeter.

Have students use grid paper or a geoboard and rubber bands to demonstrate how to count units to find lengths of sides. Locate opposite sides.

Compare rectangles. Increase by one unit the length. What happens to the perimeter? Have students demonstrate this result on a geoboard. Ask students to draw all rectangles they can that have a perimeter of 16 units.

Develop formulas for perimeter of rectangles. Encourage students to use different formulas. Insist that students give the units when expressing the perimeter of a polygon.

3. Use string and a ruler to find the circumference of a circle (model).

Have each student use a piece of string to measure the circumference of several cardboard circular regions. Then have the students lay the string along a ruler to find its measure. Help the class to see that, in each case, the circumference is about six times the radius or three times the diameter.

Each student should define circumference in his or her own words.

Research the number *pi*. Seek out the discovery of *pi*, how it is used, and its value to 15 decimal places. Display findings.

Grade 6

MEASUREMENT, continued

Objectives	Suggested Teaching Activities	Resources/Comments
	Have students draw and cut out four circular disks with diameters of 5 cm, 6 cm, 7 cm, and 8 cm. Have them roll each disk one complete turn along a meter stick and record the circumference of each disk.	
4. Use a formula to find the circumference of a circle.	Develop the relationship between the circumference and the diameter of a circle. Provide experiences in using the formula $C = \pi d$ or $C = 2\pi r$ to find the circumference of a circle. The ratio of the circumference (perimeter) of any circle to its diameter is slightly greater than 3:1. This ratio is called *pi* (π). The decimal approximation for *pi* is 3.14159. The fraction approximation is $\frac{22}{7}$. Encourage students to make a collection of various types of circular objects. Have them measure the diameter and then apply the formula to find the circumference. Students should realize that the units used to express the circumference of a circle are linear, just as those used to express perimeter are linear.	
5. By using an area unit and counting the units, find approximate area of a plane figure. (Use both metric and customary units.)	Define area as the number of square units of area found in a region. Develop the meaning of area and the method of finding the area of a rectangle by counting the units. The most basic way to show the meaning of area is to fit unit squares into a given region. Provide plastic grid paper. Students place it over figures and count the number of grid squares in the region. (Grids can be made on wax paper, plastic bags, or cellophane, drawing in the squares with black ink.) Use tiles or squares to form designs. Have students count the squares. Ask students to use 12 tiles to form figures of different shapes. Using graph paper, draw figures and find the area by counting the square units. Do the same type activity with geoboards.	

6. Find the area, using given linear measures, of a square, rectangle, triangle, circle, and parallelogram. (Use metric and customary units.)	Emphasize that area is the measure of a region. Provide examples of squares and other rectangles. Ask students to name the area of each region. Have the students cut out a rectangle. Then have them cut along a diagonal and note that the two resulting right triangles are congruent and have the same area. (Congruent triangles fit one on the other.) If possible, provide a plastic grid of square areas. Students place one over a given area and count square units. Have them divide each square in half diagonally and find the number of triangular regions in each figure. From this activity, understanding of the formula to find the area of a triangle will be developed. Encourage students to look for a pattern that suggests a method for naming each area measure. Provide activities in which they can test and revise their methods.
7. Find the surface area of prisms and pyramids. (Use metric and customary units.)	Introduce space figures and have students find the total surface area of a space figure. Models are essential for these activities. Note that when any solid is unfolded, its surface area is not changed. Explain that a prism is a special kind of polyhedron. Have students find other examples of polyhedra. Then have them make a report on the five regular polyhedra. Given a rectangular prism, what is the area of the four faces of the prism (lateral area)? What is the area of each base of the prism? What is the total area? The surface area equals the sum of the areas of the faces. Given a cube, what is the lateral area? What is the total area?
8. Find the approximate volume of a container by counting unit cubes. (Use both inch and centimeter cubes.)	Divide class into groups and provide each group with a box of cubes. Have each group arrange six cubes as a base—all different arrangements, if possible. Now have each group build a prism by adding layers to the base. They should determine the volume after each layer. Draw attention to the arrangements that form rectangular prisms and compare the volumes to the dimensions of the shape. Some students may recognize a pattern between the dimensions and the volume of a rectangular prism. (Volume = Length x Width x Height)
9. Name an appropriate metric unit to measure objects.	The metric system and metric units have been presented. Review the meaning of the prefixes. Show how the powers of 10 form the basis for the metric system.

MEASUREMENT, continued

Objectives	Suggested Teaching Activities	Resources/Comments
	Let each student weigh on a metric scale. See how many grams the student weighs. Would kilograms be better?	
	Weigh small items on a balance scale using a gram and paper clips.	
	Measure a quart of water. (Find the volume.) See if it fills a liter. Is there any left over?	
	How much does it take to fill a milliliter? A tablespoon of water?	
	Measure the temperature in Celsius and Fahrenheit. Discover the freezing point of water is 0° Celsius and the boiling point of water is 100° Celsius.	
	Set up Stations 1-4. Have directions at each station for experiments to be performed using the following materials and activities:	
	Station I— metric sticks and centimeter cubes; measure lengths of a piece of paper, eraser, table edge.	
	Station II— scales, bathroom (metric) scale, balance scale (with kilogram and gram weights), kitchen scale; measure weight of self, flour, paper clip.	
	Station III— liter and milliliter containers; measure liquid volume.	
	Station IV— Celsius scale; measure temperatures.	
	Encourage students to investigate the milligram in relation to doses of medication prescribed by doctors. Elicit from students that usually only doctors, scientists, and pharmacists are concerned with the milligram since they often work with very small quantities.	

10. Name an appropriate customary unit to measure objects.

The customary system and customary units have been presented. Let each student weigh on a customary scale. See how many pounds he or she weighs. Measure a liter of water. See if it fills a quart. Is there any left over? How much does it take to fill a milliliter? A tablespoon of water? A cup?

Measure the temperature in Fahrenheit. The freezing point of water is 32° Fahrenheit. The boiling point of water is 212° Fahrenheit.

Learning stations may be set up to provide practice. Have directions at each station for experiments to be performed using the following materials and activities:

Station I—	yardsticks and inch cubes; measure lengths of a piece of paper, eraser, table edge.
Station II—	bathroom scale, balance scale (with ounce and pound weights), and kitchen scale; measure weight of self, flour, paper clip.
Station III—	quart containers, cups, and measuring spoons; measure liquid volume.
Station IV—	Fahrenheit scale; measure temperatures.

Show the liter container and quart jar. Fill with water or sand and demonstrate that one liter is slightly more than one quart. Provide the opportunity for students to experiment with balance scales.

11. Change a given metric measure within the metric system.

Discuss with students that the metric system is based on multiples of 10. Also, discuss the meaning of the following prefixes.

centi—divided by 100
milli—divided by 1,000
kilo—times 1,000

The student must be able to arrange the metric units in order from smallest to largest and largest to smallest. EXAMPLE: millimeter, centimeter, meter, kilometer or kilometer, meter, centimeter, millimeter.

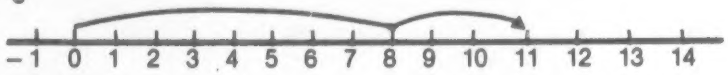
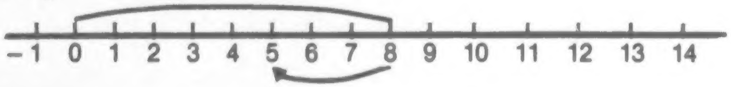
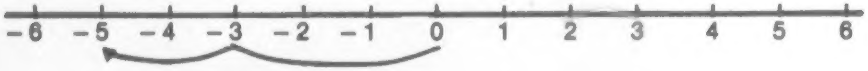
12. Solve problems using metric and customary measures.

Have students measure, record, and perhaps graph actual data to be used in problem situations.

Have students create their own problems based on measured data.

Grade 6

Unit Title: INTEGERS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Locate positive and negative integers on a number line and name the coordinate of a given point.	Two problems should be confronted: 1. Number lines can be vertical or horizontal; negative integers appear on both kinds. The thermometer will help here. 2. Sometimes students do not understand that counting from both sides of the number line starts at 0. Of course, – 1 and + 1 are next to zero because they are both one space from zero, on the opposite sides of zero.	
2. Write a set of integers in order from least to greatest or greatest to least.	The number line is the essential tool for this lesson. it is easy for students to differentiate the larger of two positive numbers; when using the number line, students begin to see that the larger number is always on the right. Another way to think of the ordering relationship is that for any two different numbers, a positive number would always have to be added to the smaller number to obtain the larger number. EXAMPLES: $2 + 3 = 5$ $-7 + 9 = 2$ $-8 + 3 = -5$ $2 < 5$ $-7 < 2$ $-8 < -5$	
3. Write the sum of two positive integers, two negative integers, or a positive and a negative integer.	Start this activity with the basic facts (sums, numbers less than 20). Then let the students apply these rules to sums with greater numbers. A. 8 + 3  B. 8 + – 3  C. – 3 + – 2 	

89A.

.89B

4. Find the additive inverse of a number.

Discuss with the class items from a newspaper concerning integers such as the temperature 25 below zero. Discuss their meanings.

Have the class draw a number line with the whole numbers 1 though 10. Discuss the term negative number and the symbol (−) and have them attach these one at a time (through − 10) to their number line.

Discuss the term *opposite.* Ask what the opposite of addition is. (Subtraction.) Ask what the opposite of adding 5 is. (Subtracting 5.) Ask what the opposite of subtracting 7 is. (Adding 7.) Why are these opposites? (Because you return to the place you started from.)

Have students choose a whole number on the number line such as 5. How far is it from zero? (5 units.) What must be done to return to zero? (Move back 5 places.) Find − 7, starting at zero. What must be done to return to zero? (Add 7.)

Point out that finding the additive inverse of a number is finding its opposite on the number line.

5. Write the difference between two positive integers, a positive and a negative integer, or two negative integers.

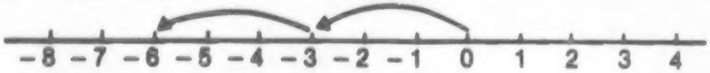
Use the number line and demonstrate the subtraction of two positive integers, a positive and a negative integer, and two negative integers.

Stories involving bills and checks help students understand the concept of addition and subtraction of positive and negative numbers.

6. Write the product of two integers.

To help students understand what happens when multiplying, first show repeated addition. From these illustrations, have students observe patterns and develop rules for multiplication of integers.

2 x − 3 =



Grade 6

Unit Title: CHARTS AND GRAPHS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Identify and interpret a bar graph, a line graph, or a picture graph by answering questions about the graph.	Have many graphs on display. Use several for oral questions. Student graphs may be used. Students should improve their ability to read graphs by investigating how graphs are constructed and discussing why a certain type graph was chosen to furnish information.	
	If a graph is to be of any value, what must it do? Does it provide a clear picture of the information? Where do we look to determine the kind of information that is included in the graph? Where is the descriptive title? What is the horizontal scale? The vertical scale? Why are two scales necessary? What information does each supply? How do you read the graph?	
	(Teaching students how to make reasonable approximations will demand more teacher time and explanation than any other skill in reading graphs.)	
	Is it easy to determine the highest point? The lowest? Can you see relationships and trends in the information in the graph?	
2. Graph a set of data using a bar graph, a picture graph, or a line graph.	Give the students some experience with constructing line, bar, and picture graphs so that their ability to read and interpret graphs will improve. Stress organization and neatness.	
	Help students label each scale and locate the points on the grid for their line graphs. Caution them to use a straightedge to draw the segments of the graph.	
	Students may need assistance with finding an appropriate title. Tell them it is standard practice to put units of time on the horizontal scale. Alert students that they must plan the amount that each space represents on the vertical scale.	
	On bar graphs, remind students to locate the longest bar at the top and to have the bars decrease in size from top to bottom. Help students conclude that an appropriate scale is one that will not crowd all the bars into the first two spaces.	
	Stress that graphs should be attractive. Color adds eye appeal. Speed in constructing a graph serves no useful purpose.	

	Dividing the class into three groups may be helpful. Let those in each group draw one of the three types of graphs. Display the graphs. Let each student explain his or her own graph or answer questions about it. Have students gather their own data on some topic and graph it.
3. Answer questions about the data on a circle graph.	Students should learn that circle graphs are used to show the division of total amounts, the relationships among the parts, and the relation of each part to the whole. Do not assume that all students will know how the sectors of the circle graph are filled. Explain that the largest sector represents the largest amount, the second largest sector represents the second largest amount, etc. Demonstrate by drawing and filling in a circle graph from a set of data. The correct method to use in circle graphs is to draw a radius at 12:00 and then enter the sectors in decreasing size in a clockwise direction.
4. Tell how a graph or chart can be misinterpreted.	Many graphs are based on a scale such as percent (from 0 to 100). But because of the space it takes to print the whole graph, newspapers, TV, and other media often only show the top of the graph (50% to 100%). The visual interpretation of the graph is then meaningless without using the labeling. There are numerous other ways in which data can be displayed and be misleading, but the importance here is to show students it takes more than glancing at a graph to interpret it.
5. Name coordinates of given points when given a graph in a plane.	Play Battleship. If a student can identify the coordinates of the point on which a ship is located, it is captured.
6. Graph, given ordered pairs of numbers.	Draw a grid representing blocks in the area around your school, locating the school in the lower left-hand corner. Ask students to locate houses, businesses, etc., specifying the number of blocks east and number of blocks north. They can then locate the ordered pair of numbers that locate each point. Use a list of ordered pairs that make a diagram when graphed. Use the index and grid on a highway map to locate certain points of interest.

Grade 6

Unit Title: GEOMETRY

Objectives	Suggested Teaching Activities	Resources/Comments
1. From a set of four-sided polygons, identify each one by name and give their distinguishing characteristics.	Have models of quadrilaterals on display. (A distinction must be made between a geometric figure and the figure's region.)	
	Classify figures as either a trapezoid, a parallelogram, a rhombus, a rectangle, a square, or a quadrilateral. Discuss the meaning of parallel and congruent.	
	Discuss the properties of each quadrilateral Use geoboards to demonstrate, and have students show the kinds of polygons.	
	Match drawings of quadrilaterals with their proper names. Match terms with their definitions.	
	Ask students to draw pictures of each of the quadrilaterals discussed in the lesson and to list the characteristics of each one. These lists may be displayed.	
	Ask students to name the opposite sides, parallel sides, and congruent sides of each figure.	
2. Identify a triangle as isosceles, scalene, or equilateral.	Have students compare the lengths of the sides of a triangle.	
	Define the three types of triangles. Classify triangles by sides as isosceles, scalene, or equilateral. Have many examples displayed. Ask students to draw triangles on the chalkboard. Then ask others to classify them according to their sides.	
	Ask students to give another name for an equilateral triangle. Point out that, according to the definitions, every equilateral triangle is also an isosceles triangle since at least two of its sides are the same length.	
	Have students write definitions for each term as it might appear in a dictionary. Practice using precise language as applied to mathematics.	
	Have students draw three triangles and make up true-false questions about each one. Then have the students answer each others' questions.	

3. Identify similar and congruent figures.	Review the concepts of similarity and congruence. Give students practice in matching many kinds of congruent shapes. Have students discuss applications of similarity (e.g., making scale models) and congruence (e.g., using patterns in manufacturing).
4. Recognize and/or reproduce a translation image of a given figure.	Discuss translation as *sliding*. Use concrete objects to represent translation images. (This activity may be done using the overhead projector.) Have students compare and contrast the translation images to the original figure. Show several examples and nonexamples of translation images of a given figure. Have students designate which examples are translations and which are not. Encourage discussion of the criteria used to distinguish translations from nontranslations. Have students produce translation images on a coordinate grid by adding a constant to each first coordinate, each second coordinate, or both.
5. Recognize and/or reproduce a reflection image of a given figure.	Have students produce a reflection image by folding a sheet of paper in some place and tracing the original figure from one side of the fold onto the other side. The fold will be the line of reflection. Have students compare and contrast the properties of the original figure to those of the image. Have students produce reflection images on a coordinate grid by changing the coordinates of the original points (e.g., negate the y component to reflect over the x axis, etc.).
6. Recognize and/or reproduce a rotation image of a given figure.	Have students produce a rotation image of a figure by turning a sheet of paper about a pin or pencil point inserted into the paper. The point where the pin holds the paper will be the point of rotation. Have students discuss what properties of the original figure remain the same in the image of the figure (e.g., size and shape) and what properties have been changed.

GEOMETRY, continued

Objectives	Suggested Teaching Activities	Resources/Comments
7. Identify the models of a rectangular prism, cone, cylinder, sphere, cube, and pyramid.	Show class models of space figures and have the students identify them. Discuss the shape of the base of each figure. How many faces? What is the shape of each face? How many vertices? How many edges? Have some or all members of the class make drawings. Cut out patterns (previously produced) of the figures, and make models. Discuss and display the models. Have students find pictures and models of figures in their immediate environment. Make a display. Help students to understand the relationship between geometry and the physical world. Bring the physical environment of the class into the discussion, and provide the students with a feeling for three-dimensional objects, not overlooking those depicted by two-dimensional drawings.	

Unit Title: PROBABILITY AND STATISTICS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Using a sample space for an event such as tossing a coin, gather data and make a graph displaying the possible outcomes.	The students can work in groups of two. Each will have a coin. List the possible outcomes: **Coin 1** **Coin 2** H H Both heads H T #1 heads; #2 tails T H #1 tails; #2 heads T T Both tails Each student will toss the coin a given number of times while the other student records or tallies the results on a frequency chart. Have students make a frequency distribution (rank the totals) and make a bar graph to show the results.	
2. Use fractions to represent the probability of an event.	Using activities similar to the preceding one, have students then record the probability of a specific event: The probability of one head and one tail (if the coins are different denominations and can be distinguished) would be $\frac{2}{4}$.	
3. Find the average of a set of numbers.	Finding the center of all the number facts is one way to find what the typical or average number is. Look for the *measures of central tendency.* Two of these measures are the *mean* and the *median.* On the chalkboard, write the set of numerals representing a set of test scores. Ask students to arrange the scores in order from greatest to least. The median test score will be the one that falls in the middle of the list. Ask students to identify the median score. Discuss the average mean score as the score that would have been made if the points had been spread evenly over all the tests. A method for arriving at this score can be illustrated with unequal piles of beans or blocks. To distribute the blocks evenly among the piles, they may be combined in one pile (addition) and then divided evenly among the original number of piles (division). Lead students to the realization that this technique will work with test scores or any other type of information to be averaged. Let students think up examples and demonstrate on the board. Some types of data used might be batting averages, game scores, daily attendance, etc.	

Grade 7

Grade 7

Unit Title: NUMERATION

Objectives	Suggested Teaching Activities	Resources/Comments
1. Compare number systems to illustrate the advantage of a place-value system.	Have students research other number systems such as Egyptian, Roman, Babylonian, etc., and compare these systems to a place-value system.	
2. Interpret the place values of whole numbers and decimals.	Review place value by having students identify what various digits mean in numbers used in everyday situations such as work with population census, accounting figures, etc. Have students complete a place-value chart such as the following:	

hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones (units)		tenths	hundredths	thousandths	ten-thousandths
	8	7	3	9	2	0	1	4	.	3	9	2	0

Objectives	Suggested Teaching Activities	Resources/Comments
	Practice reading and writing numbers correctly.	
3. Round whole numbers and decimals.	Emphasize a need for rounding through practical situations such as determining approximations of square yardage, unit pricing, total amounts of bills, and if computation results are within reason. Review rounding procedures. Encourage students to estimate by giving them about two minutes to solve 10 to 15 multistep problems using all the operations. Give exercises where the estimated answer is not the same as the true answer rounded. An example is 29 x 29. Rounding to the nearest 10 and then multiplying gives 900. Multiplying and then rounding gives 800. Calculators may be used for this activity.	

4. Read and write in standard form numerals with as many as 10 digits (whole numbers and decimals).	Review place value concepts and expanded notation. Encourage students to find large numbers from magazines, newspapers, science material, and other reference books to make bulletin boards or other displays.	
5. Compare and order whole numbers and decimals.	Discuss terms and symbols for greater than, less than, and equal to ($>$, $<$, $=$). Use real or practical problem situations involving comparison or ordering.	
6. Use scientific notation to write very large and very small numbers.	Review exponents and exponential notation, emphasizing powers of 10. Introduce scientific notation by first discussing astronomy. Use charts showing distances of planets and stars from the earth. Let students observe the large numbers, then explain scientific notation and the need for expressing large numbers in a shortened form. Use sizes of atoms, molecules, bacteria, etc. for very small numbers.	

Unit Title: OPERATIONS WITH WHOLE NUMBERS AND DECIMALS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Illustrate the use of the commutative, associative, and identity properties of addition and multiplication and the distributive property, using grouping symbols.	Begin by having students add and multiply numbers in different ways to get the same answer. Review the properties of addition and multiplication. Write equations on the board and have students identify the properties used. For example, commutative: 12 x 3 = 3 x 12; associative: (4.3 + .6) + 18 = 4.3 + (.6 + 18); distributive: 4 x 36 = 4 (30 + 6) = (4 x 30) + (4 x 6).	
2. Add and subtract whole numbers and decimals.	Calculators can be used to reinforce concepts and computational procedures. In most situations involving adding and subtracting, the calculator could be used to verify the sum or difference of columns of digits. This activity helps students apply and understand the concepts of addition, subtraction, and place value. Have students fill in missing numbers (calculator may be used). For example: $\begin{array}{r} 27 \\ 4\square \\ +\square 5 \\ \hline 130 \end{array}$ $\qquad$ $\begin{array}{r} 3,7\square 5 \\ -\square\square 46 \\ \hline 1,279 \end{array}$	
3. Solve equations involving addition and subtraction and the relation between these operations.	Stress the concept that an equation is like a balance: If you add to one side you must add to the other. Give examples such as: 5 = 10 ÷ 2 5 + 3 = (10 ÷ 2) + 3 Discuss inverse operations as undoing. To undo adding, subtract; to undo subtracting, add. A balance, either homemade or commercial, is a good tool. Examples include exercises such as: 12 + ☐ = 38 $\qquad$ 21 = 58 − ☐	

4. Multiply whole numbers and decimals.

Review the concept of multiplication as repeated addition. Practice the multiplication process in a variety of ways, for example:

Find the missing numbers.

$$\begin{array}{r} 43 \\ \times\ \ 39 \\ \hline 3\,87 \\ \square\square\square \\ \hline 1,677 \end{array}$$

Draw a line through mistakes.

$$\begin{array}{r} 547 \\ \times\ \ 52 \\ \hline 1,094 \\ 2705 \\ \hline 28,144 \end{array}$$

Calculators may be used in checking, estimating, determining partial products, etc.

5. Divide whole numbers and decimals.

Review the concept of division as repeated subtraction. Practice the division process in a variety of ways, for example:

Fill in the missing numbers.

$$\begin{array}{r} 66 \\ 35\,\overline{)\,2,340} \\ \underline{\square\square\square} \\ \square\square0 \\ \underline{\square\square\square} \\ \square\square \end{array}$$

Divide, using only the "−" key of a calculator (or divide by subtracting only).

Divide, using only the "+" key of a calculator.

Write a problem such as $.8\overline{)7.2}$ on the board. Rewrite the division problem in fractional form $\frac{7.2}{.8}$. Use the concept of equivalent fractions to illustrate why the decimal is moved.

For example, $\frac{7.2}{.8} = \frac{72}{8}$ because $\frac{7.2}{.8} \times 1 = \frac{7.2}{.8} \times \frac{10}{10} = \frac{72}{8}$

6. Solve equations involving multiplication and division and the relation between these equations.

Emphasize inverse operations as undoing. To undo multiplying, divide; to undo dividing, multiply.

Examples include exercises such as:

$4 \times 9 = \square \times 18 \qquad 48 \div \square = 8$

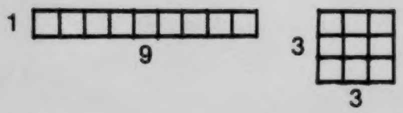
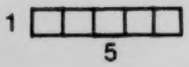
Give an expression like 2 x 2 x 2 x 2 and show the abbreviated form 2^4. Explain 2 is called the *base* (number used as a factor), and 4 is called the *exponent* (number of times the base is used as a factor).

OPERATIONS WITH WHOLE NUMBERS AND DECIMALS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
7. Find a^b when a and b are whole numbers.	Have students multiply 2 x 2 x 2 x 2 to see what 2^4 equals. Give more examples of this type. Give the abbreviated form 5^2. Show the factored form 5 x 5 and that $5^2 = 5 \times 5 = 25$. Thus $5^2 = 25$. Give students more problems of this type to solve. Show that any base number (except zero) to the exponent of zero is equal to one. EXAMPLE: $5^0 = 1$ $B^0 = 1$ Use problems that could involve exponential notation such as: If you fold a piece of paper in half five times, how many squares will you have? How many squares will you have if you fold it 15 times? $2^1 = 2$; $2^2 = 4$; $2^3 = 8$; $2^4 = 16$; $2^5 = 32$; etc.	
8. Use rounding skills to estimate the answer to a problem involving addition, subtraction, multiplication, or division of whole numbers and decimals.	Use problem situations of a practical nature such as: If you made $58.71 a week working at Al's Hamburgers, approximately how much would you make in a year? Calculators may be used for this activity.	
9. Solve one- and two-step word problems involving addition, subtraction, multiplication, and division of whole numbers and decimals.	Guide students through a strategy for solving problems. A strategy could be as follows: 1. Read the problem to determine the facts. 2. Determine what question is asked. 3. Decide what operations are needed and the order, if more than one operation is used. 4. Find the answer. 5. Ask if the solution is reasonable.	

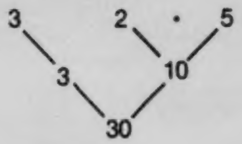
	Have students work in groups. Discuss what a two-step problem is. Have each student create four problems, each having two steps. Allow students to discuss their problems within groups to determine if there is unnecessary information or not enough information. Let each group choose their four best problems to be included on a class exercise sheet.
10. Solve multiplication and division problems involving very large and very small numbers, using scientific notation.	Review exponents and exponential notation. Use information relevant to science or social studies units (population statistics, microscopic measurements, solar measurements, etc.). Have students compose original questions about this information and challenge other students to find the answers.

Unit Title: FACTORING

Objectives	Suggested Teaching Activities	Resources/Comments
1. Find the factors of a number.	Write a sentence on the board such as □x□ = 18. Have students find all possible true replacements for the two □s. The true replacements 1, 2, 3, 6, 9, 18 are called the *factors* of 18. Use a calculator to decide whether one number is a factor of a second number. For example, is 5,538 a factor of 1,988,142?	
2. Classify numbers less than 100 as prime or composite.	Show prime numbers geometrically by arranging squares into rectangular shapes. For example, nine squares can be arranged as follows:  Notice that since there are two possible arrangements, then the number is not prime. (It has more than two factors.) Now arrange five squares:  There is only one way to arrange them. The number 5 has *exactly* two factors, 1 and 5. A number with exactly two factors is called a prime number. A number with more than two different factors is a composite number. Have students research and report on the Greek mathematician Eratosthenes' method of finding prime numbers (the Sieve of Eratosthenes). Have students make their own Sieve. To do this, make a table of whole numbers, omitting 0 and 1. Ring the first number (2). Mark out all the multiples of 2. Ring the next number not marked out (3). Mark out all the multiples of 3 that are not already marked. Continue this process until all the numbers on the table have a ring or have been marked out. The numbers with a ring are prime numbers. The numbers crossed out are composite numbers.	

3. Find the prime factorization of a number less than 100.

Use factor trees to illustrate prime factorization such as:



4. Find the greatest common factor of two numbers.

Demonstrate the definition by having students list all the factors of two numbers, choosing the common factors, and then choosing the greatest. For example:

	All Factors	Common Factors	Greatest Common Factor
12	{1,2,3,4,6,12}	{1,2,4}	4
20	{1,2,4,5,10,20}		

Use prime factorization to find the GCF of two numbers.

Write the Prime Factorization	Choose their Common Factors	The Product of these Factors is the GCF
$30 = 2 \cdot 3 \cdot 5$	$30 = 2 \cdot \boxed{3} \cdot \boxed{5}$	$3 \cdot 5 = \boxed{15}$
$75 = 3 \cdot 5 \cdot 5$	$75 = 5 \cdot \boxed{3} \cdot \boxed{5}$	
	$\downarrow$ $\downarrow$ 3 5	

5. Find the least common multiple of two numbers.

Demonstrate the definition by having students list some multiples of both numbers, choosing the multiples common to both numbers, and then choosing the smallest non-zero number from the list of common multiples. For example:

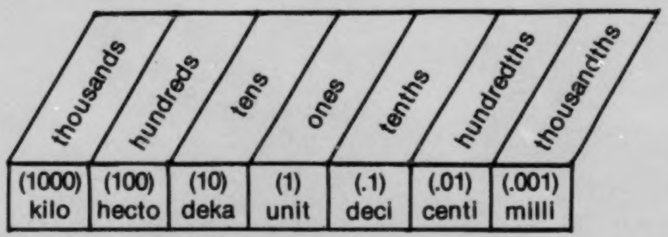
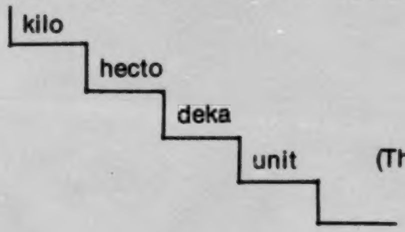
	All Multiples	Common Multiples	Least Common Multiple
3	{0,3,6,9,12,15,18,24,. . .}	{0,12,24,. . .}	$\boxed{12}$
4	{0,4,8,12,16,20,24,. . .}		

Use prime factorization to find the LCM of two numbers.

Write the Prime Factorization	Choose the Highest Power of Every Prime	The Product of these Factors is the LCM
$10 = 2 \cdot 5$	$2^2 \cdot 3^2 \cdot 5$	$2^2 \cdot 3^2 \cdot 5 = \boxed{180}$
$36 = 2^2 \cdot 3^2$		

Grade 7

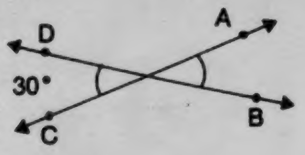
Unit Title: THE METRIC SYSTEM

Objectives	Suggested Teaching Activities	Resources/Comments
1. Measure lengths using a metric ruler or meter stick.	Measure objects within and outside the classroom. Have students record measures, using correct symbols.	
2. Round measures to the nearest centimeter and millimeter.	Have students make actual measures to the different degrees of accuracy.	
3. Estimate the length of an object to the nearest centimeter.	Have students find parts of their body that are approximate lengths. For example, 10 centimeters or 1 dekameter—the width of a hand; 1 centimeter—the width of the little finger; etc. Students may then use these body reference units to make estimates.	
4. Use the prefixes deci, centi, milli, kilo, deka, hecto.	Compare the metric prefixes to the decimal (base 10) place-value systems of numbers.  thousands (1000) kilo hundreds (100) hecto tens (10) deka ones (1) unit tenths (.1) deci hundredths (.01) centi thousandths (.001) milli	
5. Convert linear units within the metric system by multiplying or dividing by powers of 10.	When changing from a larger unit to a smaller, multiply by 10 for each step on the scale. For example: 3 km = 3000 m (from kilo down three steps—hecto, deka, and unit) 3 x 10 x 10 x 10 (or 3 x 10^3).  kilo hecto deka unit (The unit may be meter, liter, or gram.)	

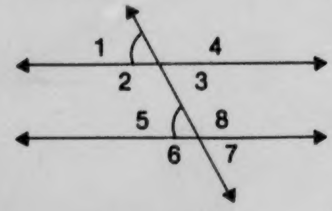
	A shorter method is to move the decimal one place to the right for each power of 10 multiplied (down the staircase) and one place to the left for each power of 10 divided (up the staircase).
6. Use square centimeters as units of area.	Have students measure and estimate area by covering drawings with plastic centimeter grid paper and counting.
7. Use a centimeter cube as a unit of volume.	Have students approximate the volume of small containers by filling them with centimeter cubes. Model volume in different ways by stacking a fixed number of cubes in different ways.
8. Use liters and milliliters as units of capacity.	Have students find common containers measured in liters or milliliters such as soft drink bottles, medicine droppers, etc. Have students do actual measuring using liters and milliliters. Compare liters and milliliters to cubic measure: $1\ \text{L} = 1\ \text{dm}^3$ $1\ \text{mL} = 1\ \text{cm}^3$
9. Use grams and kilograms as units of mass/weight.	Have students measure the mass of various objects by using balances.
10. Relate units of mass to units of volume.	The gram has about the same mass as one cubic centimeter (or one milliliter) of water. Have students test this by actual measure. Similarly, the kilogram has about the same mass as one cubic decimeter (or one liter) of water and the metric ton has about the same mass as one cubic meter of water.

Grade 7

Unit Title: GEOMETRY AND MEASUREMENT

Objectives	Suggested Teaching Activities	Resources/Comments
1. Identify, represent, and use correct symbols to name a point, line, ray, angle, segment, and plane.	Have students identify models of geometric figures within the classroom such as walls for planes or corners for angles.	
2. Use a protractor to measure angles and to draw angles of given measures.	Have students measure enough angles of different sizes to develop a thorough understanding of the make-up of a protractor. Have them use the protractor to construct various angles.	
3. Classify angles by their measures.	Students may use protractors, if necessary, to classify angles by their measurements. Between 0° and 90° : *acute* 90° : *right* Between 90 ° and 180° : *obtuse* Two angles with a sum of 90°: *complementary* Two angles with a sum of 180°: *supplementary*	
4. Identify parallel lines, intersecting lines, and perpendicular lines.	Have students find parallel, intersecting, and perpendicular lines in the classroom. Identify physical illustrations such as railroad tracks, etc. Develop the fact that vertical angles are equal, starting with one given angle and using the idea of supplementary angles. 	

Have students determine whether or not lines are parallel through corresponding angles formed by an intersecting line. Corresponding angles have the same measure.



$\angle 1 \cong \angle 5$ $\angle 4 \cong \angle 8$

$\angle 2 \cong \angle 6$ $\angle 3 \cong \angle 7$

5. Identify a polygon and its parts.

Have students observe and identify various polygonal shapes around the room and in everyday life.

6. Classify polygons according to the number of their sides.

Polygons can be classified according to the number of their sides as follows:

triangle	3	heptagon	7
quadrilateral	4	octagon	8
pentagon	5	nonagon	9
hexagon	6	decagon	10

7. Classify quadrilaterals according to their properties.

Give students several quadrilaterals, such as a rectangle, a parallelogram, a rhombus, and an isosceles trapezoid. Have them measure the angles and sides. Lead them to the properties, such as:

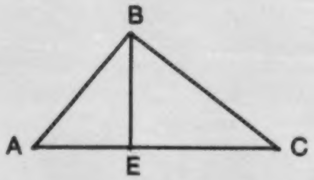
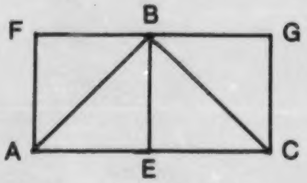
- A rectangle has four right angles and two pairs of congruent sides. (Opposite sides are parallel.)
- A parallelogram has two pairs of congruent sides and opposite angles have the same measure. (Opposite sides are parallel).
- A rhombus has four sides that are congruent to each other, and opposite angles have the same measure. (Opposite sides are parallel.)
- An isosceles trapezoid has two congruent angles less than 90° and two congruent angles greater than 90°. It has one pair of congruent sides.

GEOMETRY AND MEASUREMENT, continued

Objectives	Suggested Teaching Activities	Resources/Comments
8. Investigate the properties of regular polygons.	Stress the terms vertex, vertex angle, and central angle. Have students complete the following information on regular polygons: • Number of sides • Length of each side • Perimeter • Measure of central angle • Sum of measures of all central angles • Measure of vertex angle • Sum of measures of all vertex angles	
9. Classify triangles according to the size of their angles or sides.	Triangles can be classified as follows: • scalene—sides of different length • isosceles—at least two congruent sides • equilateral—all sides congruent • acute—all angles less than 90° • right—one right angle • obtuse—one obtuse angle	
10. Demonstrate that the sum of the interior angles of a triangle is 180°.	Have students measure the angles of various triangles using a protractor and add them. They will discover that the angles sum to 180° (with some measurement error variation). Students may use this fact to find the measures of an unknown angle in a triangle.	
11. Identify a pair of similar triangles, list the congruent angles, and write the equal ratios.	Develop the idea of similarity by using tangram pieces. Ask students if they can suggest examples of figures in everyday life that appear to be similar. Discuss scale models and scale drawings to illustrate proportional sides.	
12. Recognize two figures that are congruent.	Prepare models of rectangles, triangles, and other geometric figures. Show figures that are congruent by superimposing one upon another to show that they match exactly. Show students pairs of figures. Have them check the figures for congruency by tracing.	

	Have students draw congruent figures using graph paper.
13. Draw lines of symmetry.	Demonstrate a line of symmetry by using a large model of an isosceles trapezoid. Fold it along a line connecting the midpoints of the two bases. The two halves of the model match exactly. The line is the line of symmetry. Show that no other fold will give two congruent halves. Have students cut out various figures and fold them along lines of symmetry.
14. Relate reflections to symmetry.	The concept of symmetry can be developed through reflection. Using a mirror, the reflection in a mirror placed on an axis of symmetry leaves the design unchanged. The reflection in a mirror not placed on an axis of symmetry changes the design.
15. Find the perimeter of a polygon by measuring or using a given measure. (Measures should be both metric and customary.)	Ask students to find the distance around their desks. Explain that this is the perimeter of the desk. Give students many different shaped objects and have them find the perimeters.
16. Identify a circle and its parts.	Students should be able to identify a circle, radius, diameter, circumference, semicircle, and chord.
17. Find the circumference of a circle by measuring with a ruler and a string.	Have several circles on hand: for example, coffee can lids, margarine tub lids, and some circles, perhaps cut from posterboard, which are larger. Permanently mark the center and a diameter. Have on the table, or in your learning center, a ruler and some string. Ask students if they can measure the distance around the circle with the measuring tools on hand.
18. Determine that the value of *pi* (π) is the ratio of the circumference to the diameter of a circle.	Have students measure the circumference and diameter of several circles using string and a ruler. Ask students to compute the ratio of the circumference to the diameter of each circle. Students can discover that the ratios are all about the same. The ratio of circumference to diameter (C to d) is named by the Greek letter π (*pi*), $\frac{C}{d} = \pi$. This leads to the formula for the circumference of a circle, $C = \pi d$.
19. Find the areas of geometric regions by counting square units of measure (metric and customary).	Use plastic grid paper to cover the region or place square units over the region and count.

GEOMETRY AND MEASUREMENT, continued

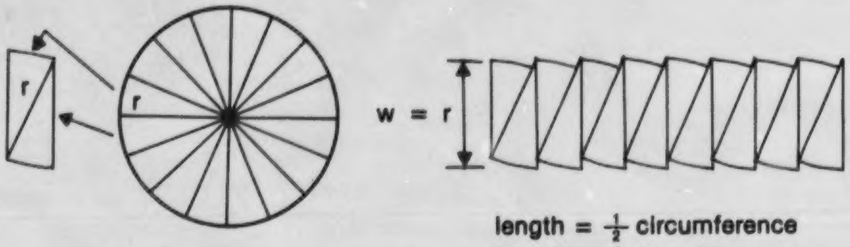
Objectives	Suggested Teaching Activities	Resources/Comments
20. Find the area of a rectangular region.	Show students rectangular regions drawn on a grid. Have students determine the length and width and then the number of square units of the area. Ask the students to study the relationship between length, width, and area of each region. Use the formula for area, A = l x w, to find the areas of various rectangular regions.	
21. Find the area of triangular region.	Show students a rectangular region and have students find the area. Draw a diagonal to form two right triangles. Guide students to observe that the area of a right triangular region is one-half the product of the sides forming the right angles. Draw a triangle such as $\triangle ABC$ below, and have students identify the base $\overline{AC}$. Draw $\overline{BE}$ and tell the class that it is perpendicular to the base and is called the height of $\triangle ABC$.  Draw a rectangle around $\triangle ABC$ with $\overline{AC}$ as the base and B on the opposite side, as below.  Lead students to observe that the area of $\triangle ABC$ is one-half the area of rectangular region AEBF plus one-half the area of rectangular region ECGB.	

Lead students to observe that if we let the base of the triangle be called b and the height h, then the area of the triangular region is $\frac{1}{2}$ bh.

Use the formula $A = \frac{1}{2}bh$ to find the area of various triangles.

22. Find the area of a circular region.

Show students a circular disk that is marked so that it can be cut into congruent parts. Cut and rearrange the parts to form a "rectangle." Guide students to see that the base of the "rectangle" is one-half the circumference, $\frac{1}{2}C = \frac{1}{2}(2\pi r) = \pi r$, and the height is r. Hence, the area is $\pi r \times r$, or πr^2.



length = $\frac{1}{2}$ circumference
length = $\frac{1}{2}(2\pi r) = \pi r$

23. Identify and compare three-dimensional figures.

Show students models of polyhedrons, especially prisms and pyramids. Show models of cylinders, cones, and spheres. Let students examine the figures and identify edges, faces, bases, and vertices. Note that three-dimensional models with all flat faces are called polyhedrons.

Have students make models of three-dimensional figures.

24. Find the surface area of prisms and cylinders.

Look at models of prisms and cylinders. Ask students how many square units of paper would be needed to cover the figures.

Show students models of prisms and cylinders that can be folded and unfolded. Guide students to find that the surface area is the sum of the areas of all the faces.

25. Find the volume of geometric regions by counting cubic units of measure (metric and customary).

Have students measure the volume of various containers by filling them with cubic units and counting.

Guide students to see that figures can have the same volume but different shapes by having them arrange cubes in different ways.

GEOMETRY AND MEASUREMENT, continued

Objectives	Suggested Teaching Activities	Resources/Comments
26. Solve problems with geometric applications using metric and customary measures.	Expose students to as many everyday problem situations as possible. Have students measure, record, and perhaps graph actual data to be used in problem situations. Have students research everyday situations in which problems involving measures occur. Examine problems that have actually occurred in city planning, construction, automobile manufacturing, etc. Have students create their own problems based on measured data.	
27. Add, subtract, multiply, and divide with denominate numbers.	A denominate number is a number that specifies a quantity in terms of measure, such as 7 inches, 3 pounds, or 5 liters. Addition, subtraction, multiplication, or division of denominate numbers can involve reducing the numbers to the same denomination and then proceeding as with ordinary numbers.	
28. Solve problems involving distances using maps and scale drawings.	Use the students' knowledge of measurement and ratio to solve problems using the distance scale found on maps. Have them plan trips by finding the shortest route, the longest route, etc.	

Unit Title: FRACTIONS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Locate and name points on a number line to represent fractional numbers.	Provide each student with an unmarked fractional number line, or simply have students make their own. A large number line in view of all students would help them as they work. Practice plotting several fractions on the number line, discussing how the point is determined; also, practice reading points on the number line. Descriptions using stores, school, home, etc. might create class interest.	
2. Write an equivalent fraction for a given fraction.	Using pie pieces or rectangular pieces cut into equivalent sizes, place or have students place six $\frac{1}{8}$ pie pieces together. Ask students to find how many $\frac{1}{4}$ pie pieces cover the six-eighths. Do several problems of this type, asking for predictions. After several times, students should be able to predict. Ask for a procedure to follow in working without the pie pieces. **Procedure or Activity:** Have students write the multiples (or times tables) of numbers 1 through 12 on long strips of paper or tongue depressors. 0 1 2 3 4 5 6 7 8 9 10 11 12 0 2 4 6 8 10 12 14 16 18 20 22 24 0 3 6 9 12 15 18 21 24 27 30 33 36 Cut out in strips so all multiples are on one strip. Give students exercises to find equivalent fractions. EXAMPLE: For $\frac{1}{3}$, take the 1 strip and the 3 strip and place over each other. Students will then have *all* multiples. Have them determine several, then ask for the procedure *not* using strips. Come back to strips to check answers.	

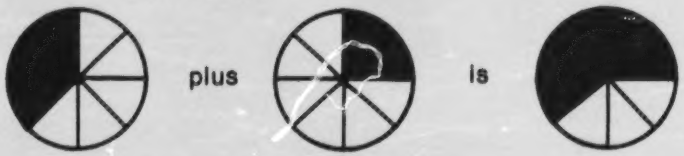
FRACTIONS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
3. Determine which of two fractions is greater.	With like denominators, students will need little or no help comparing. (Simply compare numerators.) With unlike denominators, students can make equivalent rules and compare fractions visually. Change fractions to equivalent fractions with like denominators to compare size. The technique of cross-multiplying will help students order fractions. Example: $\frac{5}{6}$ and $\frac{7}{8}$, which is larger? Multiply 8 × 5. Put 40 over 5 in the numerator. Multiply 6 × 7. Put 42 over 7. Then $\frac{5}{6} < \frac{7}{8}$ and $\frac{7}{8}$ is larger. Explain that this method is really finding equivalent fractions with the product of the given denominators as a common denominator.	
4. Find a third fraction between two given fractions (density property).	Discuss with students the concept of the *density property of rational numbers* by letting them give the teacher two fractions. Then a student will name a fraction between them. Do several until students are convinced of this property. Have students change the given fractions to fractions with like denominators to find a fraction between. For example, $\frac{3}{5}$ and $\frac{7}{8}$ are equivalent to $\frac{24}{40}$ and $\frac{35}{40}$: $\frac{25}{40}$ is between. Divide the class into two teams. Write three fractions on the board or overhead projector. The first team to answer correctly which fraction is between the other two fractions wins a point. EXAMPLES: 1. $\frac{1}{2}, \frac{3}{4}, \frac{2}{3}$; $\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{3}{4}$ 2. $\frac{-1}{2}, \frac{-1}{3}, \frac{-3}{8}$: $\frac{-3}{8}$ is between $\frac{-1}{2}$ and $\frac{-1}{3}$ These examples could be used as a worksheet.	

5. Find a terminating or repeating decimal for a fraction.	Review the definition of the fraction as a division problem. $\frac{8}{4}$ means 8 ÷ 4. Review changing improper fractions to mixed numbers. Present some common fractions students already know as decimals. EXAMPLE: $\frac{1}{2}$ = .50 and so on. Show a technique to obtain these decimals, using the well-known fractions. (Use money preferably.) Have students do their own on less well-known fractions. (You might have part of the division problem already done for them and let them finish, or use calculators.)
6. Write the fractional equivalent of a terminating decimal numeral.	EXAMPLES: 1. Given .05, read as "5 hundredths" and write as $\frac{5}{100}$; then rewrite it in lowest terms $\frac{5}{100} = \frac{1}{20}$. 2. Given 1.32, read as "1 and 32 hundredths" and write as $1\frac{32}{100}$; then rewrite in lowest terms (if desired): $1\frac{32}{100} = 1\frac{8}{25}$.

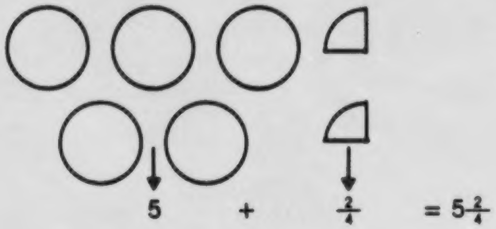
Grade 7

Unit Title: OPERATIONS WITH FRACTIONS

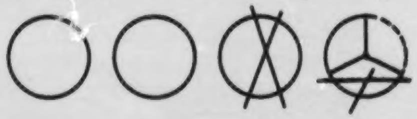
Objectives	Suggested Teaching Activities	Resources/Comments
1. Add or subtract fractions with like denominators.	Use models to introduce this concept. For example, show students circles or rectangles divided into eighths as follows:  $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$ (This procedure may be followed with different fractional parts shaded, guiding students to see and understand the rule for adding like fractions.) Follow a similar procedure for subtracting.	
2. Add or subtract fractions with unlike denominators.	Present two fractions with unlike denominators. Use models of rectangles, circles, or equal pieces. For example, cut one orange in half and one in fourths. Have students give the total if you take one half and put it with three of the fourths. Help students to see that the half must be cut in two—making fourths. (Use the other half of the orange and cut it to show that each part is now $\frac{1}{4}$.) Use the prime number method to find common denominators and point out that the common denominator is the same as the least common multiple of the two numbers. Help students to look for the common denominator as being one of the denominators. In $\frac{1}{4} + \frac{1}{8}$, 8 is the common denominator.	

3. Add or subtract mixed numbers.

Using models or pictures, demonstrate addition and subtraction. For example, $3\frac{1}{4} + 2\frac{1}{4}$. Because $3\frac{1}{4} = 3 + \frac{1}{4}$ and $2\frac{1}{4} = 2 + \frac{1}{4}$, the order property of addition allows one to add the fractional parts, then the whole parts, and then find the total.



For subtraction, model problems such as $3\frac{2}{3} - 1\frac{1}{3}$



$3\frac{2}{3} - 1\frac{1}{3} = 2\frac{1}{3}$

When students are comfortable with these subtraction manipulations, move to problems that require regrouping.

For unlike denominators in the mixed numerals, the process is the same after a common denominator has been found for the fractional parts.

Students should work many problems with pictures or models before relying completely on symbols.

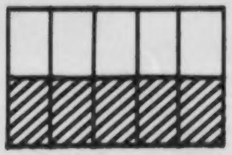
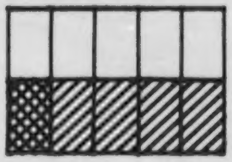
4. Solve word problems involving the addition or subtraction of fractions or mixed numbers (word problems, applications, problem situations).

Follow a strategy for solving problems, as given in Objective 9, Operations with Whole Numbers and Decimals, page 58.

Find practical problem situations. One source of practical problems involving fractions is problems involving measures. For example, finding how many yards of fencing are needed, etc. A second source of problem-solving material for adding and subtracting fractional numbers is the stock market page from a local newspaper. For example, if a certain stock sold for $87\frac{5}{8}$ yesterday and gained $1\frac{1}{4}$ points today, what is the closing price today?

Grade 7

OPERATIONS WITH FRACTIONS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
5. Multiply two fractions.	On dot paper or centimeter paper, section off a rectangular area such as a 5 × 2 rectangle. Have students shade $\frac{1}{2}$ of the rectangle. Write the question "$\frac{1}{2}$ of 10 = how many blocks?"  $\frac{1}{2}$ of $\frac{10}{1} = \frac{5}{1}$ Now shade in $\frac{1}{5}$ of the previously shaded portion so that the area shaded twice is 1 block. Write this procedure as $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$.  $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$ Do several problems until students can predict outcomes. This procedure works best if you select unit fractions.	
6. Multiply a mixed number times a whole number, a fraction, or another mixed number.	As an introduction, demonstrate to the class (if it is accelerated or average) why mixed numbers are changed to improper fractions before multiplying. EXAMPLE: $5\frac{1}{4} \bullet \frac{2}{3} = (5 + \frac{1}{4}) \bullet \frac{2}{3} = \frac{5}{1} \bullet \frac{2}{3} + \frac{1}{4} \bullet \frac{2}{3} = \frac{10}{3} + \frac{2}{12}$ The student would have to find a common denominator to determine the final product. $\frac{40}{12} + \frac{2}{12} = \frac{42}{12} = 3\frac{6}{12}$ or $3\frac{1}{2}$	

Introduce method of changing mixed numbers to improper fractions to demonstrate to students that this method is less time consuming than using partial products.

EXAMPLE: $5\frac{1}{4} \bullet \frac{2}{3} = \frac{21}{4} \bullet \frac{2}{3} = \frac{42}{12} = 3\frac{6}{12}$ or $3\frac{1}{2}$

Class Game: Divide class into teams. Put problems on board or overhead. First team member to raise hand and answer correctly wins a point.

Fraction Baseball—using multiplication of mixed numbers.

Fraction Bingo—could be made up with the answer on the bingo card. Caller would call out problems such as $3\frac{1}{2} \bullet \frac{2}{5}$. Make problem relatively easy to facilitate speed in computing.

7. Solve word problems involving the multiplication of fractions.

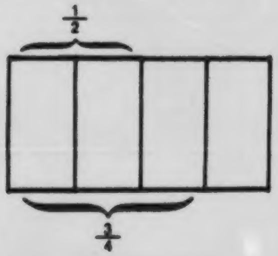
Follow a strategy for solving problems, as given in Objective 9, Operations with Whole Numbers and Decimals, page 58.

Use practical problem situations such as problems involving measures. For example, finding how many square yards of carpet are needed or finding the total square footage of a house.

8. Illustrate the meaning of division by a fraction.

This concept may be illustrated by asking "How many?" For example, how many 2s are in 8? (4 $\longrightarrow$ $8 \div 2 = 4$). Similarly, how many $\frac{1}{2}$s are in 8? (16 $\longrightarrow$ $8 \div \frac{1}{2} = 16$) Use a picture as follows:

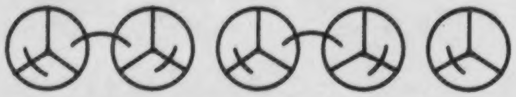
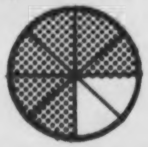
How many $\frac{1}{2}$s are in $\frac{3}{4}$? There are one and one-half $\frac{1}{2}$s in $\frac{3}{4}$.



$\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$

Grade 7

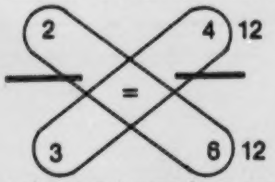
OPERATIONS WITH FRACTIONS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
9. Use models to divide whole numbers by fractions.	For example, $5 \div \frac{2}{3}$. How many $\frac{2}{3}$s are in 5?  (Note that two parts of each circle is one $\frac{2}{3}$ and one part is half of $\frac{2}{3}$.) There are seven $\frac{2}{3}$s and one-half $\frac{2}{3}$. Hence, there are seven and one-half $\frac{2}{3}$s in 5, or $5 \div \frac{2}{3} = 7\frac{1}{2}$.	
10. Use models to divide fractions by fractions.	For example, $\frac{3}{4} \div \frac{5}{8}$. How many $\frac{5}{8}$s are in $\frac{3}{4}$?  Note that $\frac{3}{4}$ is the shaded portion of the circle and that five of the small sections is one $\frac{5}{8}$. Also note that one section is $\frac{1}{5}$ of the $\frac{5}{8}$. Hence, there is one $\frac{5}{8}$ in the $\frac{3}{4}$ with $\frac{1}{5}$ of $\frac{5}{8}$ left. Therefore, there are one and one-fifth $\frac{5}{8}$s in $\frac{3}{4}$, or $\frac{3}{4} \div \frac{5}{8} = 1\frac{1}{5}$.	
11. Use an algorithm for dividing with fractions.	Review the reciprocal of a number. Ask students how many 4s are in 16? ($16 \div 4 = 4$); how many $\frac{1}{2}$s are in 16? ($16 \div \frac{1}{2} = 32$). Note that $16 \times 2 = 32$ and that 2 and $\frac{1}{2}$ are reciprocals. Guide students to observe that dividing a number by $\frac{1}{2}$ is the same as multiplying by its reciprocal 2. Use other examples, such as $18 \div \frac{1}{3}$. Lead to the algorithm: to divide by a number *n*, multiply by the reciprocal of *n*.	

12. Divide two fractions or mixed numbers.	Review changing mixed numbers to equivalent improper fractions. After mixed numbers are changed to improper fractions, the division process is the same as above.
13. Solve problems involving the division of fractions or mixed numbers (word problems, applications, problem situations).	Follow a strategy for solving problems, as given in Objective 9, Operations with Whole Numbers and Decimals, page 58. Find practical problem situations. Problems involving perimeter, area, or other measures offer good sources.

Grade 7

Unit Title: RATIO AND PROPORTION

Objectives	Suggested Teaching Activities	Resources/Comments
1. Write ratios to describe rates or compare quantities.	Have students find examples of ratios in the supermarket or other stores. For example: If three cans of soup sell for 89¢, the ratio would be $\frac{3}{89}$, read as three for 89. Ratios may be used to compare various quantities such as U.S.-made cars to imported cars in the parking lot. If there were 87 U.S.-made cars and 27 imported cars, the ratio would be $\frac{87}{27}$, read as 87 to 27.	
2. Illustrate equivalent ratios.	Note that a ratio is a comparison of two numbers by division, so any facts about the division operation apply to ratios. Emphasize that ratios are not fractions but comparisons of two numbers. Fraction concepts can be used to find equivalent ratios. Review the processes for finding equivalent fractions. Demonstrate equivalent ratios by giving students a rate such as three pieces of candy for 20¢. Ask students to write the same rate with different numbers (i.e., six pieces for 40¢).	
3. Write two equivalent ratios as a proportion.	Discuss the meaning of proportion as a sentence stating that two ratios are equivalent. A proporation such as $\frac{2}{5} = \frac{6}{15}$ is read, "2 to 5 is equivalent to 6 to 15."	
4. Find an unknown term in a proportion.	Find unknown terms by using equivalent fractions. $\frac{5}{6} = \frac{n}{24}$ (5 is to 6 as n is to 24) Since $\frac{5}{6} = \frac{20}{24}$, n = 20 Guide students to discover that in true proportions, cross products are equal.  Use cross products to find unknown terms in a proportion.	

5. Write and solve a proportion for appropriate problem situations.	Follow a strategy for solving problems, as given in Objective 9, Operations with Whole Numbers and Decimals, page 58. Good problem sources can come from price rates, scale drawings, measure problems, recipe changes, etc.	

Grade 7

Unit Title: PERCENT

Objectives	Suggested Teaching Activities	Resources/Comments
1. Write fractions and decimals as hundredths and as percents, and write a percent as a decimal and as a fraction.	Students should learn to recognize a value three ways: $0.25 = \frac{25}{100} = 25\%$, $\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$, $90\% = \frac{90}{100} = 0.90$	
2. Write ratios as percents and percents as ratios.	On the board, write ratios such as $\frac{3}{5}$, $\frac{7}{25}$, $\frac{1}{2}$, etc. Have students write a ratio equivalent to each of these having a second term of 100. Students may use the proportions $\frac{3}{5} = \frac{n}{100}$, $\frac{7}{25} = \frac{n}{100}$, $\frac{1}{2} = \frac{n}{100}$, etc. Note that a ratio whose second term is 100 is called a percent. $\frac{3}{5} = \frac{60}{100} = 60\%$, $\frac{1}{2} = \frac{50}{100} = 50\%$	
3. Find what percent one number is of another number.	Students can use ratios or decimals to solve percent problems. For example, to find what percent 12 is of 48: **proportion** $\frac{12}{48} = \frac{n}{100}$ $12 \times 100 = 48 \times n$ $\frac{1200}{48} = n$ $25 = n$ $\frac{12}{48} = \frac{25}{100} = 25\%$ **decimals** ☐% of 48 = 12 $p \times 48 = 12$ $p = \frac{12}{48}$ $p = 0.25$ $p = 25\%$	
4. Find a percentage of a number.	Use ratios and decimals to find percentage. For example, what is 9% of 347. **proportion** $\frac{n}{347} = \frac{9}{100}$ $n \times 100 = 347 \times 9$ $n = \frac{3123}{100}$ $n = 31.23$ **decimals** $9\% \text{ of } 347 = N$ $\frac{9}{100} \times 347 = N$ $0.09 \times 347 = N$ $31.23 = N$	

5. Find a number when a percent of it is known.	Use ratios and decimals. For example, 17% of some number is 10.54. What is the number?

proportion	**decimals**
$\frac{10.54}{n} = \frac{17}{100}$	17% of N = 10.54
10.54 x 100 = 17 x n	$\frac{17}{100}$ × N = 10.54
$\frac{1054}{17} = n$	0.17 x N = 10.54
62 = n	N = 62

6. Solve problems involving percent (including problems involving personal finance situations such as interest on loans, discounts, etc.).	Follow a strategy for solving problems, as given in Objective 9, Operations with Whole Numbers and Decimals, page 58. Demonstrate by interpreting several examples of word problems, and guide students to interpret for themselves. After problems are interpreted, have students find the solutions. Have students interpret problems by writing the correct proportion. Ask students to develop word problems of their own, including the types of problems as in Objectives 3, 4, and 5.

Grade 7

Unit Title: GEOMETRIC CONSTRUCTIONS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Construct a segment, an angle, and a triangle congruent to a given segment, angle, and triangle.	Make sure students can identify each of the figures. Emphasize that in constructions with a ruler and compass, no measurements are made with the ruler, and a protractor is not used for measuring angles.	
2. Construct a line perpendicular or parallel to a given line.	After students have completed their constructions, allow them to measure angles with a protractor to check for accuracy.	
3. Construct a bisector of a given line segment and a given angle.	Have students demonstrate different constructions on the chalkboard to show steps involved.	
4. Construct an equilateral triangle, a regular hexagon, a square, and a regular octagon within a circle.	Ask students to name objects whose forms look like regular polygons, such as buildings, signs, tiles, etc. Have students design a bulletin board using constructions of regular polygons.	
5. Find special geometric relationships by drawing and measurement.	Guide students to discover geometric relationships by having them make careful measurements. Have students discover the answer to questions such as: • What kind of figure is formed when the midpoints of a quadrilateral are connected in order? (parallelogram) • Connect the midpoints of two adjacent sides of a triangle. How does the length compare with the length of the other side? ($\frac{1}{2}$) • Extend the sides of several regular polygons and measure the angles. What do you find? (Angles are the same.) • Construct lines and draw a line through them. Measure the angles formed. What do you find? (Corresponding angles are equal.)	

Unit Title: INTEGERS

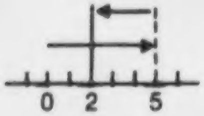
Objectives	Suggested Teaching Activities	Resources/Comments
1. Locate positive and negative integers on a number line and name the coordinate of a given point.	A good visual aid would be a teacher-made thermometer. Most students have a fairly good intuitive idea about the location of points on a thermometer. Two problems should be confronted. • Number lines can be vertical or horizontal, and negative integers appear on both kinds. The thermometer will help here. • The other problem is that sometimes students do not understand that counting for both sides of the number line starts at 0. Of course, − 1, and + 1, are next to zero because they both are one space from zero, on opposite sides of zero. Everything starts from one central spot—the winds of the earth, the strands of one's hair, etc. This concept is a good one for students to think about when working with integers.	
2. Write a set of integers in order from smallest to largest or largest to smallest.	The number line is the essential tool for this lesson. It is easy for students to differentiate the larger of two positive numbers; when using the number line, students begin to see that the larger number is always on the right. Another way to think of the ordering relationship is that for any two different numbers, a positive number would always have to be added to the smaller number to obtain the larger number. EXAMPLE: $-8 + 3 = -5$ $2 + 3 = 5$ $-7 + 9 = 2$ $-8 < -5$ $2 < 5$ $-7 < 2$ Make cards showing numbers like − 14, − 13, − 12....... − 1, 1, 2, 3, 4........ 12, 13, 14, 15. Use all numbers except zero. Put numbers at random on four or five different colored cards so that the students are automatically divided into color-coded teams. Then, call out a particular subset of numbers. Examples: All multiples of 5, all negatives greater than − 7 and less than − 13, all odd numbers, etc. The students who qualify come to the front. Have them line up together at a designated spot. Place the card with a zero on it somewhere along the chalk tray. Students will arrange themselves in the proper order.	

INTEGERS, continued

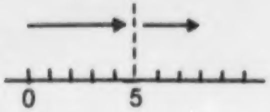
Objectives	Suggested Teaching Activities	Resources/Comments
3. Add two positive integers, two negative integers, or a positive and a negative integer.	One way to help students with this topic is to substitute for *positive* and *negative* something more easily visualized. For example, use blocks to represent positive numbers and marbles to represent negative numbers. One block and one marble would cancel each other; then rules for adding positive and negative numbers should be developed. A semi-abstract way to do the same thing is to let "in the black" stand for positive and "in the red" stand for negative. If students understand the concepts, they are able to become more efficient in their thinking and in their expression of results. It is imperative to be strict and careful about use of words *positive* and *negative* as opposed to *plus* and *minus.* Another useful tool is upraised symbols for positive and negative symbols. This will limit future confusion.	
4. Find the additive inverse (opposite) of a number.	Review the idea of *inverse.* Students know operations have opposites. They need to extend this idea to realize the inverse of numbers. Two rational numbers are additive inverses of each other if the sum of the two numbers is zero. Use stories like the following: Suppose you take a trip of five miles to the east of your house (for positive). How would you get home? Stress that *additive inverse* and *opposite* are the same. It is useful to develop the idea of why we need an additive inverse. This development may be done with the use of a balance.	
5. Subtract two positive integers, two negative integers, or a positive and a negative integer.	Develop the defintion of subtraction as the inverse of addition. • Have students find an integer to make an equivalent addition statement from a subtraction statement. EXAMPLE: Given: 5 − 3 = 2 and 5 + ? = 2. Answer: − 3. This example shows the subtraction and equivalent addition statement. • Show the graphing of subtraction to average and above average students. Emphasize the graphing of subtraction as the opposite of how addition is graphed.	

EXAMPLE:

5 + − 3



5 − − 3 is opposite



- Use the number line to show subtraction as the difference between the coordinates of two points or the directed distance between two points.

6. Multiply two positive integers, two negative integers, or a positive and a negative integer.

Emphasize that multiplication of whole numbers is repeated addition. Break several multiplication problems down into addition such as 5 x 3 = 3 + 3 + 3 + 3 + 3. Have students work several until they are proficient. Introduce a positive times a negative and break down to an addition statement. Example: 5 x (− 3) = (− 3) + (− 3) + (− 3) + (− 3) + (− 3) = − 15. Use the commutative property to find answer to − 5 x 3, same as 3 x (− 5) or (− 5) + (− 5) + (− 5). Illustrate on a number line, showing repeated addition of negative numbers. Practice several until students discover the rule for a negative by using opposites: a negative times a negative must be the opposite of a negative times a positive.

A game of Concentration using integers would be a good activity.

7. Divide two positive integers, two negative integers, or a positive and a negative integer.

Review the fact that multiplication and division of whole numbers are inverse operations. The same is true for integers: multiplication and division of integers are inverse operations.

Show students multiplication sentences and have them write related division sentences. For example:

multiplication	**division**
24 = 3 x 8	24 ÷ 3 = 8
60 = − 2 x (− 30)	60 ÷ (− 2) = − 30
− 16 = − 8 x 2	− 16 ÷ (− 8) = 2

8. Solve equations involving integers.

Review the methods of using inverse operations to solve equations.

EXAMPLE: − 5 + x = 2.

Inverse operation: − 5 + x + 5 = 2 + 5 or x = 7.

INTEGERS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
	Review the additive and multiplication inverse properties. EXAMPLE: $5 + (--5) = 0$ $5 \bullet \frac{1}{5} = 1$ Stress the concept that an equation is like a balance: If you add to one side you must add to the other. Give examples such as: $5 = 10 \div 2$ $5 + 3 = 10 \div 2 + 3$ Use additive and multiplication inverse properties to solve: EXAMPLE: $-5 + x = 2$ $\begin{array}{rcr} -5 + x &=& 2 \\ +5 & & +5 \\ x &=& 7 \end{array}$ For subtraction, use the rule: $a - b = a + (-b)$ EXAMPLE: $2 - 5 = 2 + (-5) = -3$ A balance, either homemade or commercial, is a good tool. Be careful to include problems where trial and error become impossible. Try having students draw up flow charts for solutions to their problems. Guide students to discover the rule for dividing by positive and negative integers.	
9. Solve problems involving integers (word problems, applications, problem situations).	Follow a strategy for solving problems, as given in Objecitve 9, Operations with Whole Numbers and Decimals, page 58. When possible, use practical problem situations such as bookkeeping, etc.	

10. Solve inequalities involving integers.

Ask students to find the solution of an equation such as $y - 3 = 4$. Write the inequalities $y > +7$ and $y < +7$ on the chalkboard. Ask students for solutions. Note that the equation has one solution, but the inequalities have an infinite number of solutions.

Demonstrate that inequalities are solved in the same way as equations, but the solution to an inequality can be more than one number.

Illustrate the solution to inequalities by graphing. Find practical situations for using inequalities.

EXAMPLE: Present a graph of a stress equation. Discuss information that can be read from the graph. (A beam will not give way for variables less than the stress line values.)

Grade 7

Unit Title: COORDINATE GEOMETRY

Objectives	Suggested Teaching Activities	Resources/Comments
1. Use integers and fractions to name coordinates of points in a plane and graph points in a plane when the coordinates are given.	Have students form groups. Have each group make a straight-line drawing of their own design on graph paper and determine the coordinates of the strategic points and instructions for drawing the lines. Have groups exchange instructions and try to reproduce the drawings.	
2. Use a coordinate system to locate points on a map.	Allow students to work with different maps or copies of maps to locate cities or places given map coordinates.	
3. Graph a figure similar to a given figure by using larger graph paper or by multiplying the coordinates.	Review the concept of similar figures, recalling that in similar triangles the matching angles are congruent and the ratios of the lengths of matching sides are equal. Have students name everyday examples of similar figures. Have students graph several figures once on regular graph paper and again on larger graph paper. Measure sides and angles to check relationship. Repeat by multiplying coordinates.	
4. Produce the reflection image of a figure by graphing.	Review the concept of symmetry and symmetric figures by having students fold paper and make cuts from the fold to produce symmetric figures. Also, mirrors may be used when talking about a reflection image. Have students identify and graph the reflection image of various points.	
5. Complete a table for a given equation and graph the number pairs.	A good equation to start with is the distance equation. That is, distance equals rate times time ($d = r \bullet t$). If the rate is 20 km/hr, this equation would be $d = 20 \bullet t$. Find the distance in km for several different times in hours to complete a table. Use the coordinates from the table to graph the equation $d = 20t$.	

t	1	2	3	4	5
d	20	40	60	80	100

Unit Title: PROBABILITY AND STATISTICS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Make a frequency distribution from a given set of data.	Have students keep a record of cars passing their homes according to color, make, or classification (foreign, U.S., truck, etc.) They can make a frequency distribution table from the data collected. The above may also be done in the cafeteria, recording hamburgers with mayonnaise or mustard.	
2. Find the measures of central tendency (mean, median, mode) of a set of data.	Give students a set of grades and ask them to find the mean. Be sure they understand this is an average. Collect some statistics such as height in inches or centimeters, weekly allowances, wrist measurements in millimeters. Then find the mean, median, and mode of this data as a class activity.	
3. Identify a graph as a bar graph, line graph, picture graph, or circle graph, and interpret the data by answering questions.	Have students bring a graph that they have found in a newspaper or magazine. Divide into groups, and have each group make a list of three questions about each graph. Then have the groups exchange questions and graphs to see if the answers can be found.	
4. Draw an appropriate graph to display a set of data using a bar graph, circle graph, or line graph.	Have students collect data for various assigned topics such as height of classmates, cars passing intersection per hour, etc. Decide as a class which data could be used for each kind of graph. Then use groups of students to prepare the graphs. A prize for the graph that is the neatest, prettiest, most colorful, biggest, smallest might add some fun.	
5. Construct sample spaces and use them to determine the probabilities of simple events.	Inform the class that a sample space must list all the logical outcomes for a given event. Then make a sample space for tossing a penny; a penny and a dime. Ask students to list events in the sample space for a wooden cube with different colored faces.	

Grade 8

Grade 8

Unit Title: NUMERATION

Objectives	Suggested Teaching Activities	Resources/Comments
1. Write whole numbers, decimal numbers, and integers in expanded form, using exponential notation.	Give students numbers in expanded form and have them use calculators (if available) to find the number in standard form. Review place-value concepts and expanded notation. Have students make graphs using large numbers. For example, graph populations, sales distributions, etc.	
2. Arrange, in order of size, and locate on the number line a set of rational numbers in fraction and decimal form.	Have students compare results (real or simulated clock times) of individual events in track meets or other sporting events. Organize and graph the data collected.	
3. Round whole numbers and decimals to the nearest given power of 10.	Emphasize a need for rounding through practical situations such as determining approximations of square yardage, unit pricing, total amounts of bills, and if computation results are within reason. Review rounding procedures.	
4. Find the number of significant digits in a given numeral.	Write a measurement on the chalkboard such as 150m. Assuming that the measurement is to the nearer 10 meters, ask students how many 10 meters there are (15). Note that both the 1 and the 5 are needed to tell how many units of measure there are. Assuming that the measurement is to the nearer meter, ask students how many meters there are (150). In this case the 1, the 5, and the 0 are needed to tell how many units of measure there are.	
5. Write large and small numbers in scientific notation.	Review exponents and exponential notation, emphasizing powers of 10. Introduce scientific notation by first discussing astronomy. Use charts showing distances of planets and stars from the earth. Let students observe large numbers, then explain scientific notation and the need for expressing large numbers in a shortened form. Define scientific notation as a product in which one factor is between one and 10 and the other factor is a power of 10.	

Unit Title: OPERATIONS AND PROPERTIES

Objectives	Suggested Teaching Activities	Resources/Comments
1. Estimate sums and differences.	Give students grocery store cash register tapes and have them estimate total sums. Make cards with pictures and prices of various grocery store items. Give students a short time to select a certain amount of groceries. The student getting closest to the amount wins the game.	
2. Estimate products and quotients.	Use everyday examples when possible such as unit price estimates, per capita income, per person living space, quantity buying, etc.	
3. Solve problems using numbers expressed in scientific notation.	Begin by multiplying and dividing powers of 10. Use problems involving large numbers to show the efficiency of using scientific notation, i.e., distances between stars and planets. Example problem situations include seconds in a day or year, problems involving astronomy, average income of certain populations, average consumption of certain fast foods.	
4. Evaluate an expression by using grouping symbols and following the correct order of operations.	When introducing order of operations, first put an expression such as 16 + 20 ÷ 4 − 2 on the board. Most people will say that its value is 7. However, point out that the agreed upon order of operation is: 1. multiplication and division in order from left to right 2. addition and subtraction in order from left to right Thus, 16 + 20 ÷ 4 − 2 = 19. But if 16 + 20 ÷ 4 − 2 is to mean 7, then there are symbols to signify adding 16 to 20 before dividing by 4 and then subtracting 2. Thus [(16 + 20) ÷ 4] − 2 = 7. Point out that the grouping symbols should be used whenever there is any possible doubt concerning the order of operations.	

OPERATIONS AND PROPERTIES, continued

Objectives	Suggested Teaching Activities	Resources/Comments
5. Write and find values for expressions involving rational numbers.	Review the order of operations as given above. Note that the same rules apply when evaluating expressions involving rational numbers. Have students work with actual information when possible. For example, they might compute temperature changes for certain times of the day.	
6. Solve addition and subtraction equations.	Review inverse operations of adding and subtracting. Discuss inverse operations as undoing. To undo adding, subtract; to undo subtracting, add. For example: $5 \longrightarrow 5 + 7 \longrightarrow 5 + 7 - 7 \longrightarrow 5$ Stress the concept that an equation is like a balance. A balance is a good tool for illustration. Examples include exercises such as: $N + 8.52 = 76.03$, $1230 = N - 652$	
7. Solve multiplication and division equations.	Follow a similar process as above using multiplication and division. For example, use geometric formulas for area or volume.	
8. Write a mathematical expression for a word phrase.	Have students do matching activities where they match a word phrase with a mathematical expression. For example: $x + 14 = 26$ ⟶ the sum of x and 14 is 26 $26 - x = 14$ ⟶ x less than 26 is 14 Have students write word phrases from given equations and vice versa.	
9. Use number sentences to solve problems.	Supply a variety of problems, using situations familiar to students. Include career-oriented and consumer problems. Have groups of students compose original problems to exchange with other groups. Have each group translate the problem and solve it.	

Unit Title: **MEASUREMENT**

Objectives

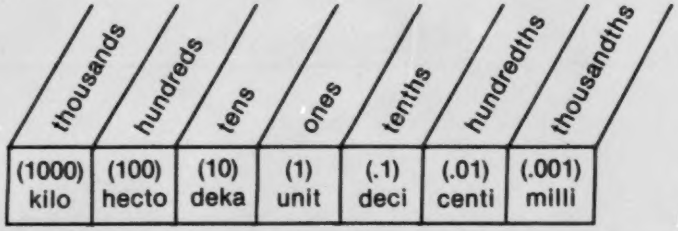
1. Estimate and measure in metric units length, mass/weight, capacity, and temperature.
2. Express one metric unit in terms of another.

Suggested Teaching Activities

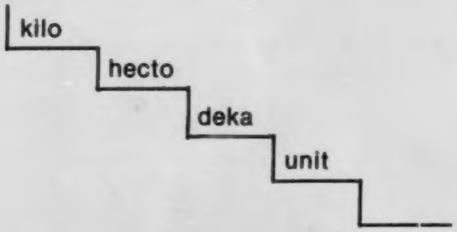
Provide a variety of experiences for students to use estimating and measuring skills. Have students measure objects to different degrees of accuracy.

Students should choose appropriate units of measure before measuring an object.

Make a chart of metric prefixes and place values. Compare the metric prefixes to the decimal (base 10) place-value system of numbers.


thousands
hundreds
tens
ones
tenths
hundredths
thousandths
(1000) kilo
(100) hecto
(10) deka
(1) unit
(.1) deci
(.01) centi
(.001) milli

When changing from a larger unit to a smaller, multiply by 10 for each step on the scale. For example: 3 km = 3000 m (from kilo down three steps—hecto, deka, and unit) 3 x 10 x 10 x 10 (or 3 x 10^3).


kilo
hecto
deka
unit

(The unit may be meter, liter, or gram.)

A shorter method is to move the decimal one place to the right for each power of 10 multiplied (down the staircase) and one place to the left for each power of 10 divided (up the staircase).

Resources/Comments

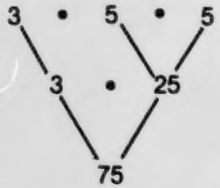
MEASUREMENT, continued

Objectives	Suggested Teaching Activities	Resources/Comments
3. Solve problems using metric units.	Have students measure, record, and perhaps graph actual data to be used in problem situations. Have students research everyday situations in which problems involving measures occur. Use problems that have actually occurred in city planning, construction, automobile manufacturing, etc. Have students create their own problems based on measured data.	
4. Solve problems involving distances using maps and scale drawings.	Use students' knowledge of measurement and ratio to solve problems using the distance scale found on maps. Have them plan trips by finding the shortest route, the most scenic route, the longest route, etc.	
5. Tell which of two measurements is more precise.	When objects are measured by two units of measure, the more precise is the measurement using the smaller unit of measure. For example, if the length of an object were measured as 3 cm and 32 mm, the more precise would be 32 mm.	
6. Determine precision from a stated measurement.	Review rounding skills. Write a measurement on the chalkboard such as 1,299.8 m. Round this number to the nearer one (1,300 m); nearer 10 (1,300 m); nearer 100 (1,300 m). Note that all three answers are the same. Ask students if they were given a measurement of 1,300 m, would it be a result of rounding to the nearer one, nearer 10, or nearer 100. Stress that they must assume that the measurement is to the nearer 100 meters. Examples: Give the precision of each measurement. 5,000 km (1,000 km) 3.0 m (0.1 m) 7.02 g (0.01 g)	

7. Find the greatest possible error of a given measurement.	Have students measure strips of poster board to the nearer centimeter. Ask how short a strip can be and still have a length of 5 cm to the nearer centimeter. (4.5 cm.) Ask how long a strip can be and still have a length of 5 cm. (5.5 cm.) Emphasize that there is a difference between a stated measure and the true measure (called the error of measurement). The greatest possible error is one-half the unit of measure.
***8. Find the relative error of a given measurement.	Measure a variety of objects using centimeters. Discuss the greatest possible error in each measurement. (0.5 cm.) Then discuss the importance of the error in each measurement, i.e., 0.5 cm error in a 35 cm measurement would be less important than a 0.5 cm error in a measurement of 6 cm. Have students use ratios to express relative error: $\frac{0.5}{36}$, $\frac{0.5}{6}$.

***enrichment

Unit Title: FACTORING

<table>
<tr><th>Objectives</th><th>Suggested Teaching Activities</th><th>Resources/Comments</th></tr>
<tr><td>1. Find the prime factorization of a given number.</td><td>Review the definition of prime numbers. Illustrate prime factorization using factor trees.
</td><td></td></tr>
<tr><td>2. Find the greatest common factor of two numbers.</td><td>Demonstrate the definition by having students list all the factors of two numbers, choosing the common factors, and then choosing the greatest.
EXAMPLE:
<table>
<tr><th>All Factors</th><th>Common Factors</th><th>Greatest Common Factor</th></tr>
<tr><td>$12\ \{1,2,3,4,6,12\}$
$20\ \{1,2,4,5,10,20\}$</td><td>$\{1,2,4\}$</td><td>4</td></tr>
</table>
Use prime factorization to find the GCF of two numbers.
EXAMPLE:
<table>
<tr><th>Write the Prime Factorization</th><th>Choose their Common Factors</th><th>The Product of these Factors is the GCF</th></tr>
<tr><td>$30 = 2 \cdot 3 \cdot 5$
$75 = 3 \cdot 5 \cdot 5$</td><td>$30 = 2 \cdot \boxed{3} \cdot \boxed{5}$
$75 = 5 \cdot \boxed{3} \cdot \boxed{5}$
$\qquad\qquad 3 \quad 5$</td><td>$3 \cdot 5 = \boxed{15}$</td></tr>
</table></td><td></td></tr>
</table>

3. Find the least common multiple of two numbers.

Demonstrate the definition by having students list some multiples of both numbers, choosing the multiples common to both numbers, and then choosing the smallest non-zero number from the list of common multiples.

EXAMPLE:

All Multiples	**Common Multiples**	**LCM**
$3 \longrightarrow \{0,3,6,9,12,15,18,24,\dots\}$	$\longrightarrow \{0,12,24,\dots\}$	12
$4 \longrightarrow \{0,4,8,12,16,20,24,\dots\}$		

Use prime factorization to find the LCM of two numbers.

EXAMPLE:

Write the Prime Factorization	**Choose the Highest Power of Every Prime**	**The Product of these Factors is the LCM**
$10 = 2 \bullet 5$	$2^2 \bullet 3^2 \bullet 5$	$2^2 \bullet 3^2 \bullet 5 = \boxed{180}$
$36 = 2^2 \bullet 3^2$		

Grade 8

Unit Title: OPERATIONS WITH INTEGERS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Write positive and negative integers for given situations.	Positive and negative numbers can be used to signify more and less, gains and losses, credits and debits, temperatures, stock market results, etc.	
2. Compare and order integers.	Have students plot integers on a number line to compare them. Lead them to discover that the greater of any pair of integers is to the right of the other on the number line.	
3. Find the absolute value of integers and evaluate expressions involving absolute value.	Draw a number line on the chalkboard and plot on it several integer values. Ask students to tell the distance each number is from zero. Note that the distance is always positive. The absolute value of an integer tells the distance the number is from zero on the number line.	
4. Find the sum or difference of two integers.	Develop the concept of adding and subtracting integers by using integers in practical problem situations such as gains and losses, credits and debits, stock market results, etc. Draw a number line and lead students to discover how it can be used to combine integers.	
5. Find the product or quotient of two integers.	To illustrate multiplying, begin by using the repeated addition concept. For example, $2 + 2 + 2 = 6 = 3 \times 2$ and $-7 + (-7) + (-7) = -21 = 3 \times (-7)$. Since multiplication is commutative, $3 \times (-7) = -7 \times (3)$. Therefore, students can conclude that: positive x positive = positive positive x negative = negative negative x positive = negative	

The product of two negatives can be illustrated by the following example:

Ask questions as each line is written.

$-3[2+(-2)] = -3(0)$ Why?

↑ is the same as

$-3(2) + (-3)(-2) = 0$ Why? (distributive)

$-6 + \text{what} = 0$

Therefore (-3) times (-2) must equal 6.

Since multiplication is the inverse operation of division, the same rules apply.

6. Use the properties to solve problems involving integers.

Review strategies for solving word problems:

1. Read the problem to determine the facts.
2. Determine what question is asked.
3. Decide what operations are needed and the order, if more than one operation is used.
4. Find the answer.
5. Ask if the solution is reasonable.

Give students a variety of examples and have them practice translating the words to numerical sentences.

Have students devise original problems for class discussion.

7. Use an integer as an exponent.

Discuss the meaning of negative exponents.

Have students find real situations in which negative exponents are useful such as writing very small numbers in scientific notation (sizes of molecules, atoms, neutrons, etc.)

Grade 8

Unit Title: RATIONAL NUMBERS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Define and identify rational numbers.	Define a rational number as any number that can be expressed as a quotient of integers, where the divisor is not 0. Have students plot various rational numbers (and their opposites) on a number line.	
2. Arrange two or more numbers expressed in $\frac{a}{b}$ form in order of size, and locate each one on the number line.	Lead students to discover that two numbers in the form $\frac{a}{b}$ can be easily compared if they have a common denominator. It is then simply a matter of comparing numerators. If the fractions do not have a common denominator, their equivalents should be written with a common denominator.	
3. Find a third rational number between any two given rational numbers (density property).	Use the average of two rationals to illustrate the density property. Note that the sets of integers and whole numbers do not possess the density property. A rational number can be placed between any two rational numbers, but an integer cannot always be placed between two integers (i.e., there is no integer between −5 and −4) and a whole number cannot always be placed between two whole numbers (i.e., there is no whole number between 1 and 2).	
4. Find a terminating or repeating decimal for a fraction and find the fractional equivalent of a terminating decimal.	Review the method for changing a fraction to a decimal. Give students examples of both terminating and repeating decimals. Ask students how they would define terminating and repeating decimals. Discuss these definitions to determine their completeness. Have students determine when a fraction can be expressed by a terminating decimal.	
5. Solve problems involving the addition or the subtraction of fractions.	Use practical problem situations whenever possible such as problems involving measures or stock market results. Emphasize that rational numbers in fractional form must be expressed as equivalent fractions with common denominators before they can be added or subtracted. Include mixed numerals in your discussion but express each as an improper fraction.	

6. Solve problems involving multiplication and division of fractions.	Use models to develop the concepts of multiplication and division of fractions. Ask students to review stock market pages and follow results of certain stocks for a period of time. Have students create original problems involving rational numbers on the stock market. Use problem situations involving measures.
7. Determine the number named by a zero and a negative power of a rational number.	Use integral exponents with decimals, especially as applied in scientific notation, and indicate the number of places the decimal point needs to be moved and the direction to be moved. Use step-by-step examples: • $.1 = 10^{-1}$ $.01 = 10^{-2}$ $.001 = 10^{-3}$ $.0001 = 10^{-4}$ and $.00001 = 10^{-5}$ • $10^{-4} = .0001$ $10^{-7} = .0000001$ • 5.0034 $5 + .003 + .0004$ $5 \times 1 + (3 \times .001) + (4 \times .0001)$ $(5 \times 10^{0}) + (3 \times 10^{-3}) + (4 \times 10^{-4})$ • $.00036 = 3.6 \times 10^{-4}$ Also, negative exponents can be used with other rational numbers: $2^{-3} = \frac{1}{2^3}$ $= \frac{1}{8}$ $(\frac{1}{2})^{-3} = \frac{1}{(\frac{1}{2})^3}$ $= \frac{1}{\frac{1}{8}}$ $= 1 \div \frac{1}{8}$ $= 1 \times 8$ $= 8$ $\frac{1}{3^{-2}} = 3^2$ $= 9$
8. Use negative exponents in scientific notation.	Illustrate the need for using negative exponents in scientific notation by considering very small measures such as measures of molecules, atoms, or other microscopic objects.
9. Solve problems involving very small numbers, using scientific notation.	Have students pose questions about information from magazines or newspapers involving small measures. Students then can use their knowledge of rational numbers and scientific notation with negative exponents to answer these questions.

Grade 8

Unit Title: RATIO, PROPORTION, AND PERCENT

Objectives	Suggested Teaching Activities	Resources/Comments
1. Write ratios to describe rates or compare quantities.	Define ratio by examples: comparison of boys to girls in a classroom, amount of flour to amount of water in making paste (or other recipe ingredients), sports records, etc. Point out that the word *rational* comes from *ratio* and ask students to make comparisons of the two concepts using their knowledge of rational numbers and ratio.	
2. Write two equivalent ratios as a proportion, and if necessary, find the unknown term.	Review the concept of equivalent fractions and have students relate this idea to equivalent ratios. Discuss proportions as an expression of equivalent ratios. Find unknown terms by using knowledge of equivalent fractions: $\frac{6}{9} = \frac{x}{18}$, $\frac{6}{9} \bullet \frac{2}{2} = \frac{12}{18}$, so $x = 12$ Using simple ratios written in proportion statements, lead students to discover the method of using cross products for solving for unknowns in more difficult proportions. $\frac{1}{2} = \frac{3}{6} \longrightarrow 1 \times 6 = 2 \times 3$, etc. Generalize to $\frac{n}{16} = \frac{8}{124} \rightarrow 124n = 8 \times 16$	
3. Solve problems involving proportion.	Have students work in small groups to set up proportions from written descriptions. For example, Raymond Junior High School has 165 students and 15 teachers. Dully Intermediate has 231 students and 21 teachers. $\frac{165}{15} = \frac{231}{21}$ Use problem situations involving recipe changes; scale drawings or scale models; school facilities available per person; rate of speed, distance and time; and unit pricing to develop skill in using proportions to solve for unknown information.	

4. Illustrate the relationship between fractions, decimals, and percents.

Review the meaning of percent ("for each 100" or "hundredths").

Have students write a rational number in three forms—fractional, decimal, and percent. By looking for patterns in these representations, students can generate methods for changing from one form to another.

5. Solve problems involving percent.

Review percents as ratios. 16% = $\frac{16}{100}$

Have students translate verbal statements involving percents into proportions. For example:

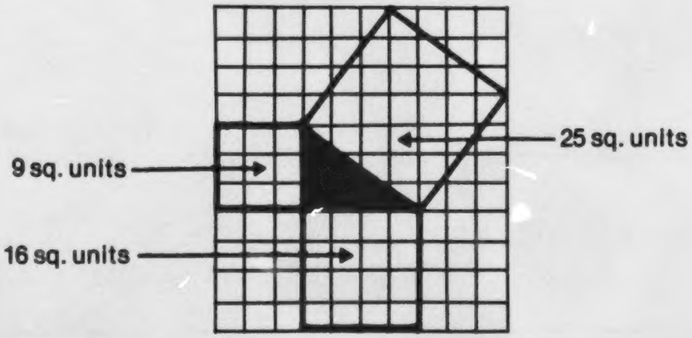
- 25% of one dozen eggs is 3 eggs: $\frac{25}{100} = \frac{3}{12}$
- 6 students in a class of 30 entered the contest because 20% of the class was allowed to go: $\frac{6}{30} = \frac{20}{100}$

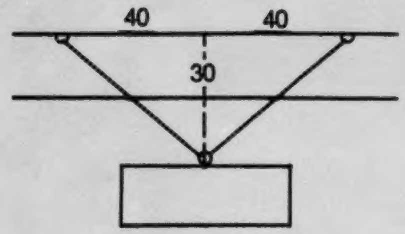
Students should use their knowledge of equivalent fractions to test the proportions for validity. When students are familiar with these translations, a part of the proportion may be an unknown:

- What percent is 15 of 32? $\frac{x}{100} = \frac{15}{32}$
- 36 is 8% of what number? $\frac{36}{x} = \frac{8}{100}$

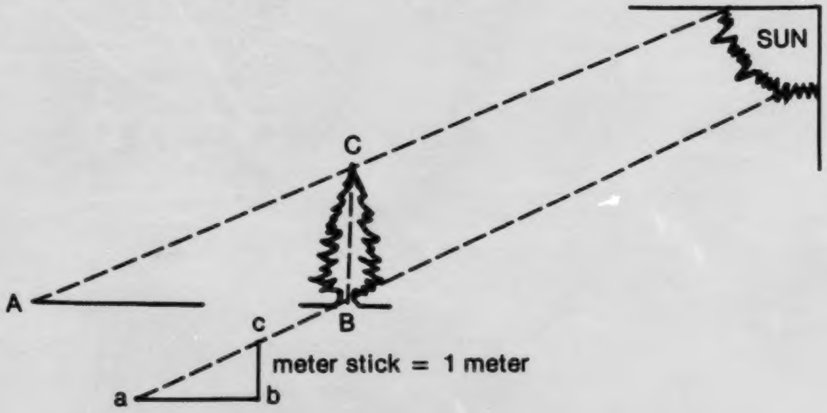
Have students use their knowledge of proportions to solve problems involving percent, e.g., discounts, simple interest, construction of circle graphs. Students can collect data and write their own problems.

Unit Title: RIGHT TRIANGLES

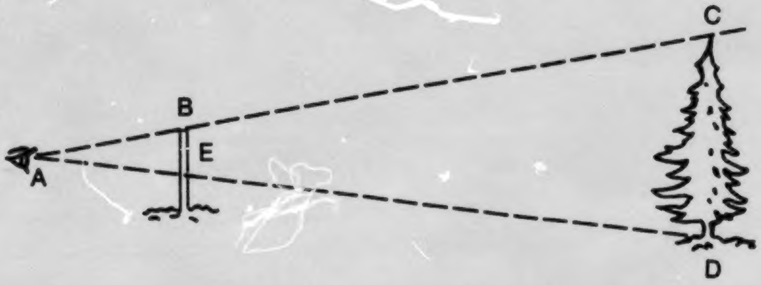
Objectives	Suggested Teaching Activities	Resources/Comments
1. Define a right triangle and identify its parts.	Discuss the characteristics of a right triangle: the right angle, the hypotenuse, the two legs. Discuss the relationship of the other angles to the right angle (both smaller) and the legs to the hypotenuse (both shorter). Have students locate right triangles in the environment. For example, a path between any two nonadjacent bases on a baseball diamond makes two right triangles, a ladder leaning against a wall, etc.	
2. Use the Pythagorean Property to find the length of the unknown side of a right triangle.	Review concepts of area, especially of squares and triangles. Use a geoboard (or graph paper on the overhead) to illustrate the Pythagorean Property by constructing squares on the legs and hypotenuse of a right triangle.  Have students find the areas of the squares on the legs and the square on the hypotenuse. Have students do their own construction using right triangles with different dimensions and compare results. Lead students to the discovery that the sum of the areas of the squares on the legs is equal to the area of the square on the hypotenuse. Using their knowledge of finding areas of squares, have students develop the equation $a^2 + b^2 = c^2$ where a, b are lengths of the legs and c is the length of the hypotenuse.	

	Have students use the geoboard or graph paper to find the missing dimension of a right triangle until they feel comfortable using the formula alone.
3. Solve problems using the Pythagorean Property.	Have students find the distance from home plate to second base on a regular baseball diamond. Have students find the length of wire needed to hang a sign 30 cm from the ceiling from two hooks 80 cm apart.  Have students look for other situations in which the Pythagorean Property would be helpful in solving problems and have them develop questions to be answered using this property. Calculators could be used to perform the computations needed in everyday situations.
4. Define similar triangles and identify their corresponding parts (for right triangles).	Review proportion concepts in the context of scale drawings and relate these concepts to the definition of similar triangles: the lengths of the sides of one triangle compared to the lengths of the corresponding sides of the other triangle are equivalent ratios (in proportion). Have students identify the corresponding parts of similar triangles, especially right triangles. Encourage students to discuss the characteristics of similar triangles, e.g., congruent corresponding angles.
5. Find the missing dimension in similar triangles by using proportions.	Review methods for solving equations involving proportions. Have students use their knowledge of proportions to determine the dimensions of a triangle similar to a pictured one, given either one side of the new triangle or the scale for the drawing.

RIGHT TRIANGLES, continued

Objectives	Suggested Teaching Activities	Resources/Comments
6. Solve problems that involve indirect measurement by using proportion and similar triangles.	Review the properties of similar triangles and proportion equations. Engage students in some outdoor activities involving indirect measurement. For example: *Shadow method of measuring heights*  Have students hold a meter stick ($\overline{cb}$) upright and measure its shadow ($\overline{ab}$). Since ΔABC and Δabc are similar, measurement of the tree's shadow ($\overline{AB}$) will allow for finding the height of the tree ($\overline{CB}$) using the proportion $\frac{\overline{CB}}{\overline{AB}} = \frac{cb}{ab}$.	

Stadia Method of Measuring Heights



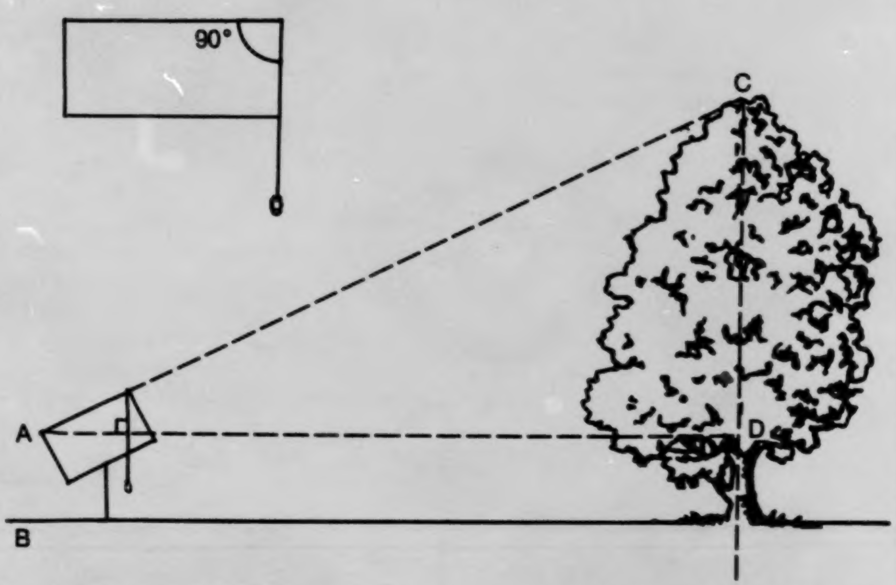
Have students sight along the meter stick at the top and bottom of the tree and locate points B and E on the meter stick. Length BE can then be read and $\overline{AE}$ and $\overline{AD}$ can be measured with a tape measure. Since $\triangle ABE$ and $\triangle ACD$ are similar, CD can be found using the proportion: $\frac{AE}{AD} = \frac{BE}{CD}$.

Have students look for other situations calling for indirect measurement. Have them illustrate the problem and write the activity for other students to do.

7. Solve problems that involve indirect measurement by using the tangent ratio.

Discuss the terms *opposite side* and *adjacent side* as applied to angles in a right triangle. Review angle measurement. Then, to develop meaning for the tangent ratio, present several similar right triangles and have the students use calculators to compute the ratio of the opposite side to the adjacent side for each of the acute angles in each of the triangles. The students should begin to notice a pattern of equivalent ratios. (Corresponding angles, or congruent angles, have equivalent ratios.) When the tangent ratio concept seems sufficiently developed, introduce students to a table of tangent values for use in solving problems.

RIGHT TRIANGLES, continued

Objectives	Suggested Teaching Activities	Resources/Comments
	Have students construct a clinometer using a 10" x 15" piece of cardboard, string, paper clip, and brad. The string is attached in the upper right hand corner using the brad. The paper clip is tied to the other end of the string to form a plumb line. A protractor (or picture of 0 to 90° arc) should be attached to the cardboard as shown below.  The students then can use the clinometer to measure the height of the tree. The student sights the top of the tree along the upper edge of the clinometer and reads the angle where the plumb line crosses the arc. Using this angle measure and the distance of the base of the object from the instrument, the height of the tree can be calculated using the tangent ratio: $\text{TAN } \angle \text{ CAD} = \frac{CD}{AD}$ Height of tree = CD + AB	

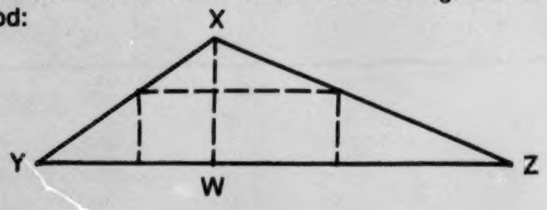
Have students look for other situations that could call for the use of the tangent ratio in indirect measurement and write problems to fit these situations (the best angle to throw a football, etc.).

8. Use the sine or cosine ratio to find the length of one side of a triangle, given the measure of the acute angle and the other leg or hypotenuse.

Develop the concepts for sine and cosine ratios in a manner similar to the tangent ratios. Students could use calculators for the computations needed to develop a pattern of values. When the basic idea of sine and cosine ratios is established, introduce the table for these values and give the students practice in reading the table to find sine values when given angle measurements and vice versa.

Present problem situations in which the measurements of an acute angle and one leg of the hypotenuse of a right triangle are given and ask students to find the measure of the other leg.

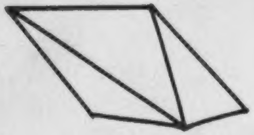
Unit Title: GEOMETRIC CONSTRUCTIONS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Find the supplement and complement of an angle.	Define supplementary and complementary angles. Point out that the pairs of angles need not be adjacent to be supplementary or complementary. Give students adequate practice in measuring angles and identifying their supplements and complements. Use practical problem situations when possible.	
2. Copy a segment, an angle, and a triangle using a compass and straight-edge.	Demonstrate each construction with an explanation of why the construction is accurate. Have students copy each step of the construction as it is done. Then give students ample practice in doing the constructions individually. Review the term *congruent* and apply to copying segments, angles, and triangles.	
3. Construct a square, right triangle, and rectangle using a compass and straight-edge.	Review constructions of parallel lines, perpendicular lines, midpoints, and bisectors. Give students a chance to use their knowledge of these polygons and the above constructions to devise a method of constructing squares, right triangles, or rectangles. Through presentation and discussion of their ideas, lead the students to determine which methods are most accurate and efficient.	
4. Find the measure of the third angle of a triangle, given the measure of the other two angles.	Have students demonstrate the sum of the interior angles of a triangle by the following method:  For any triangle XYZ, fold Y onto $\overline{YZ}$ so that $\overline{XW}$ is formed. Fold Y over to W, Z over to W, and X down to W. Students can now see that the sum of ∠ X, ∠ Y, and ∠ Z is 180°. Have students repeat this activity with other triangles and make a generalization from their results.	

Have students use the generalization to find the measure of the third angle of a triangle when given the other two measures.

5. Find the angle sum for any polygon and the measure of each angle of a regular polygon.

Discuss the sum of the interior angles in any polygon. Pose the question to students: "How could we find the sum of a polygon's angles if we had no protractor with which to measure?" Encourage students to use their knowledge of the sum of the angles in a triangle until a method is developed to subdivide the polygon into triangles.



A pentagon can be subdivided into three triangles. The sum of the angles of the pentagon, then, is
3 x 180° = 540°.

Students may notice that the number of triangles is always two less than the number of sides in the polygon.

Students can use this knowledge of the sum of the angles in any polygon to find the angle measure in a regular polygon. For example, a pentagon has an interior angle sum of 540°. Since a regular pentagon has five congruent angles, one of these angles measures $\frac{540°}{5} = 108°$.

6. Construct an equilateral triangle, regular hexagon, square, and regular octagon within a circle.

Demonstrate each construction with an explanation of why the construction works. Have students copy each step of the construction as it is done. Then give students practice in doing the constructions individually.

7. Name and sketch prisms and pyramids and identify their properties.

Classify a polygon according to the number of sides, list any of the following names that apply to it, and give a reason for each listing: acute, equilateral, isosceles, obtuse, parallelogram, rectangle, regular, rhombus, right, scalene, square, trapezoid, pentagon, hexagon, and octagon.

Follow this procedure using prisms and pyramids.

Have students collect objects and pictures of objects and classify them according to a chosen geometric attribute.

Have students use graph paper to practice sketching prisms and pyramids.

Unit Title: GRAPHING, RELATIONS, AND FUNCTIONS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Locate points on the coordinate plane (including positions on maps).	Have students discuss uses of the coordinate plane, i.e., mapping. Introduce vocabulary: *ordered pairs, horizontal axis, vertical axis, quadrants I, II, III, IV.*	
	Give students practice in locating points on a map using puzzles, games, and data collected from newspapers or magazines.	
2. Find the coordinates for a given point shown on the coordinate plane, e.g., locating positions on maps.	Give students a graph grid and have them draw or trace a simple picture on it. Then they are to name the points in sequence. Have two students exchange points in sequence to see if they can reproduce the original. If not, have them work together to find the mistake or mistakes.	
	Give students maps and have them name coordinates for given locations.	
3. Slide a figure on a graph by changing the ordered pairs.	Give students an opportunity to experiment with changing the first number in each ordered pair that describes a figure. After graphing several sets of new coordinates, students should begin to see a pattern—that changing the first number by a given amount moves the figure to the right or left and changing the second number by a given amount moves it up or down.	
	Relate this movement to slide transformations and note that size and shape are not affected and the figure does not turn.	
	Group students in pairs. One student draws two congruent figures on a grid. The second student must decide what to do to the first and second numbers of each ordered pair to move one figure onto another.	
4. Find the reflection of a given figure on the coordinate plane.	Let students experiment with changing the sign of each first number or each second number in a set of ordered pairs. Students should notice that the resulting figures are reflections of the original figures and that size and shape are not affected.	
	Lead students to form a generalization for generating reflections of figures on the coordinate plane. Provide them with opportunities to test their generalizations.	

5. Construct and complete a table of values for a linear equation.	Lead students in making a table of values that satisfy an equation. Use a problem situation to generate the equation, e.g., "Judy and Tom each caught at least one fish. Together they caught 8 fish. How many could they each have caught?" While making the table of values, students should realize that only integral values are reasonable here, but that other situations might include other than integral values. Give students practice in completing tables of values for problem situations and encourage them to discuss reasonable values (integers, negative numbers, etc.) and patterns in results (x increases as y decreases, etc.)
6. Graph ordered pairs of integers from a table.	Have students translate the pairs of numbers in the table to ordered pairs, and graph the points on the coordinate plane. Encourage students to discuss any patterns that are apparent in the graph.
7. Graph a linear equation over all real numbers.	Review the construction of tables of values and graphing ordered pairs. Draw students' attention to the problem situation involved. "Are only integral values reasonable, or are fractional values possible in the ordered pairs that satisfy the equation?" Students should begin to see the linear orientation of the graph after plotting several solution pairs and begin to realize that not every point should, or can, be plotted when graphing over all real numbers.
***8. Write an equation for a graph or a table of ordered pairs.	As an enrichment or problem-solving experience, give students a chance to write an equation that would generate a certain set of ordered pairs. Students will probably begin using trial-and-error but soon should develop questioning techniques such as "What can be done to the first number to get the second number?" Have each of two students secretly write an equation (involving one operation) and make a table of ordered pairs that are solutions to the equation. Students then exchange tables and try to generate the other student's equation from the values in the table.
9. Find the solution to a problem using the graph of a linear equation.	Use a problem situation for which students have developed a table of values and graphed the linear equation. Ask questions about the problem that require students to read the graph (for values other than those used in the table.)

***enrichment

GRAPHING, RELATIONS, AND FUNCTIONS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
10. Find the unique solution to a problem, using the intersection of two linear equations.	Introduce students to the usefulness of graphing to solve problems involving two situations (two equations). Use equations such as the following to motivate use of the graphs: John had twice as many pennies as nickels. He had 56¢ in all. How many pennies and nickels did he have? Students should develop an understanding of why the intersection of the two graphs is the solution to the problem.	
***11. Draw the graph for a linear inequality.	For students who have a strong understanding of graphing linear equalities, extend the concept to include linear inequalities.	
12. Read and interpret a curved graph.	Have students apply their knowledge of reading linear graphs to reading curved graphs. Questions about the information shown on the graph may be teacher-generated or student-generated. Graphs may be of temperature changes, acceleration rates, etc.	
13. Make a curved graph for a given equation.	Have students apply their knowledge of making tables of values for a linear equation to making tables for a nonlinear equation. Acceleration of falling objects on the moon, $d = .8t^2$, is an example of equations that may be used. Encourage students to interpret the resulting graph and make predictions.	

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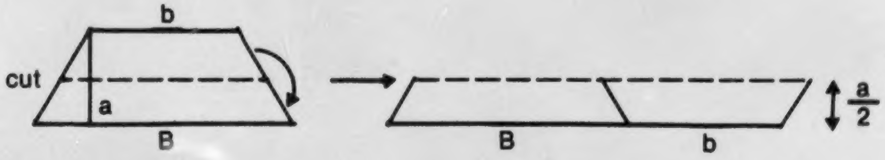
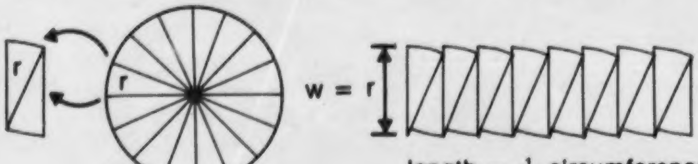
Unit Title: GEOMETRY

Objectives	Suggested Teaching Activities	Resources/Comments
1. Use models to find the areas of rectangles, parallelograms, trapezoids, and triangles.	Review the concept of area using square units to cover and count. Have students make a bulletin board (or use individual pieces of graph paper) to show different shapes covering the same area. Students could also show all rectangles with areas of 24 square units, all triangles with areas of 24 square units, etc. Encourage students to look for patterns in the dimensions and areas of the shapes to develop formulas. After students understand and can use the formula for area of a rectangle, use this concept to develop methods for finding areas of parallelograms, triangles, and trapezoids. Students can cut a triangular piece from a cardboard parallelogram and reassemble it to form a rectangle. Congruent cardboard triangles can be joined to form a parallelogram to demonstrate the relationship of the triangles' areas to the parallelogram's area. Students can cut a cardboard trapezoid in half and reassemble it to illustrate the relationship of its area to a parallelogram whose dimensions are half the original height and the sum of the two bases.  	
2. Find the surface area of a prism or pyramid, using models.	Make covers for the models from graph paper to show surface area. Relate total surface area to the area of the faces of the solid. Students should be encouraged to see surface area as the sum of the area of the faces of the solid rather than as a formula.	
3. Find the volume of a prism or cube, using models.	Have students use tagboard to construct boxes in convenient sizes (or use ready-made models) and measure the volume by filling them with cubic inch blocks or cubic centimeter blocks.	

Grade 8

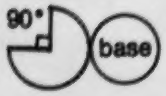
GEOMETRY, continued

Objectives	Suggested Teaching Activities	Resources/Comments
	From experience with various constructions and models, students should notice patterns in the dimensions and volumes of regular solids and should develop an understanding of possible formulas for volume.	
4. Find the volume of a pyramid.	Students may use sand in plastic models to find the relationship between the volume of a pyramid and the volume of its corresponding prism.	
5. Find the circumference and area of a circle.	Review the concept of circumference as the distance around the circle and its relationship to the radius of the circle. Show students a circular disk that is marked so that it can be cut into congruent parts. Cut and rearrange the parts to form a "rectangle." Guide the students to see that the base of the rectangle is one-half the circumference, $\frac{1}{2}C = \frac{1}{2}(2\pi r) = \pi r$, and the height is r. Hence, the area is $\pi r \times r$, or πr^2.  length = $\frac{1}{2}$ circumference length = $\frac{1}{2}(2\pi r) = \pi r$	
6. Find the surface area and volume of a cylinder.	Review the concept of surface area as the sum of the areas of the faces. Lead students to see the cylinder as two circles and a rectangle of dimensions, circumference x height. Students can demonstrate this fact by cutting a cardboard tube and flattening it into a rectangle or by using a rectangular piece of paper to form a tube. Relate the method of finding volume of a prism to finding volume of a cylinder: area of base determines how many cubes to fill one layer and the height of the cylinder determines how many layers of cubes. Students may use sand in plastic models to show that cylinders and prisms of equal heights that have bases of equal area have the same volume.	

7. Find the surface area and volume of a cone.

Use models of cylinders and cones that have the same base and height. By filling the cylinder from the cone, students can see that the volume of the cone is $\frac{1}{3}$ that of the cylinder.

Students can flatten a model of a cone to observe its surface area by cutting around the base and making a slit in the side to the top.



Then students can see that the surface area of a cone is the area of the base + $\frac{3}{4}$ the area of a circle whose radius is the length of the slit.

***8. Investigate methods for finding surface area and volume of spheres.

Give students opportunities to test their guesses of how to determine surface area and volume of spheres. Ask: "Should a sphere have a larger or smaller surface area than a cylinder of the same volume?" "Why do oil refineries build storage tanks in the shape of spheres?" and other questions that require intuitive thinking. (Do not require use of formulas.)

9. Solve problems using area and volume.

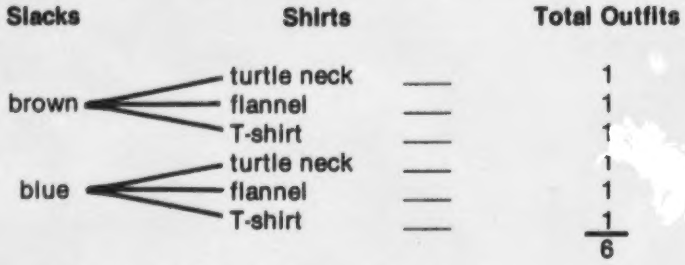
Have students use their knowledge of area and volume to solve manufacturing problems. How much metal is required to make a can? Which of two boxes are the most economical to make, giving the most volume for least surface area? Why does an insect float and a person sink in water, yet an insect's movement is more affected than a person's when wet?

Have students arrange six blocks to form solids with the most surface area, least surface area, and most volume.

Incorporate concepts of area and volume into classroom discussions and urge students to look for situations in their out-of-school activities that require knowledge of area and volume.

***enrichment

Unit Title: COUNTING

<table>
<tr><th>Objectives</th><th>Suggested Teaching Activities</th><th>Resources/Comments</th></tr>
<tr><td>1. Determine what kind of counting procedure is appropriate for a given situation.</td><td>Have students give some examples of things they can count: people in the room, points in a game, books on a shelf.
Have students give some examples of things they cannot count: stars in the sky, blades of grass in a lawn.
Lead students to an understanding of the necessity for different counting and estimating procedures.</td><td></td></tr>
<tr><td>2. Find the total number of choices by using a tree diagram.</td><td>Give students problem situations in which the number of combinations must be found. For example, putting together an outfit consisting of slacks, a shirt, and sweater with two choices of slacks, three choices of shirts.


Slacks
Shirts
Total Outfits
brown
turtle neck
flannel
T-shirt
blue
turtle neck
flannel
T-shirt
1
1
1
1
1
1
6

Extend to include three factors in the combination.</td><td></td></tr>
<tr><td>3. Find the total number of choices by multiplying.</td><td>Present several examples to guide students to see the relationship between the tree diagrams and the multiplication process.
(2 slacks with 3 sweaters ⟶ 2 groups of 3 ⟶ 2 x 3 = 6)</td><td></td></tr>
<tr><td>4. Solve problems by employing the multiplication-counting principle.</td><td>Present familiar situations that call for combination counting: Possible telephone numbers? Class schedule possibilities? Uniform combinations?
Have students develop their own combination problems and experiment with three and more factors.</td><td></td></tr>
</table>

5. Find the number of outcomes when choices are dependent.

Give students practice in distinguishing when choices are independent and when they are dependent (when they have some influence on each other).

Have students apply their knowledge of the multiplication counting principle to outcomes involving dependent choices, e.g., how many four-digit numbers can be made in which the two middle digits are identical?

6. Find the number of arrangements that can be made from a given set of objects (using factorials).

Discuss arrangements without replacement as a specific situation involving dependent choices. Use the students' knowledge of the multiplication-counting principle as applied to dependent choices to develop a method for counting possible arrangements (permutations). Students should begin to see a pattern in the factors in these situations: $n(n-1)(n-2)\ldots(1)$ and can be introduced to the use of factorials for counting.

Have students look for situations in which the factorial method of counting would be useful.

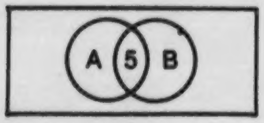
7. Read and make Venn diagrams.

Introduce the method of using Venn diagrams to represent collections of data. Together with students, diagram information gathered from the class such as how many students are in the band and how many take music lessons. Discuss the reason for representing the circles as overlapping.

Give students collected data (or have students collect the information) to diagram. Be sure to include sets that are exclusive (students 14 years of age and over, students under 14).

8. Use Venn diagrams to solve problems.

Use Venn diagrams generated during the previous activities. Have students answer questions, using the diagrams:



A = band = 20
B = music lessons = 10
Class of 35

How many students take band but not music lessons?
How many students take music lessons or band?
How many students do not take either band or music lessons? Etc.

Have students generate their own questions about diagrammed data gathered in science, social studies, and other subjects and challenge other students to use the diagrams to answer the questions.

Unit Title: PROBABILITY AND STATISTICS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Find probabilities when choices are independent or dependent.	Review the concept of counting possible outcomes when choices are independent and when they are dependent. Discuss *probability* as the likelihood of one event taking place when compared to all the possibilities. Introduce the expression of probability as a fraction: $\text{probability} = \frac{\text{number of designated outcomes}}{\text{number of possible outcomes (when all outcomes are equally likely)}}$	
2. Make predictions when given the probability of an event and use probability of a simple event to solve problems.	Use the students' knowledge of finding the probability of an event to lead them to develop a method for using the probability to predict the number of desired outcomes that will be in a given number of trials. Students may use direct multiplication or proportions. For example, if 5% of the light bulbs made are usually defective, how many probably would be defective out of a sample of 60? 5% of 60 is $.05 \times 60 = 3$ or $\frac{5}{100} = \frac{y}{60}$ and $y = 3$. Pose probability problems orally to students and have them design experiments for them. Encourage students to trace possible sources of bias and seek ways to eliminate it.	
3. Find the probability of compound events (dependent or independent).	Experiments are the best way to get the point across. An example of such an experiment is as follows: Given a box containing one marble labeled A, two labeled B, and three labeled C, draw a marble without looking and note its letter. Replace the marble and shake the box. Draw again, without looking. Give the probability of drawing: (1) A first, B second, (2) A on both draws, (3) C first, A second, etc. Have students complete a chart of possible outcomes to determine the probability.	

	After working through several situations, students may begin to see a pattern developing between the probabilities of the single events and that of the compound event. (The probability of the compound event is the product of the single probabilities.)
4. Find the probability that one outcome or another might occur.	Have students experiment as before with colored marbles or coins and then complete a chart of the possible outcomes. Lead students to see the probability of one event or the other as being the sum of their individual probabilities.
***5. Find conditional probabilities.	Allow students who show an interest in probability concepts to experiment with conditional probability questions. For example: What is the probability that a die landed on 5 if you know it landed on a number greater than 3? Have students discuss how the condition affects the list (or number) of possible outcomes.
6. Organize and present data in tabular and graphical form, e.g., bar graph, line graph, circle graph, pictograph.	Give students a set of data and have them graph it all ways listed (bar, circle, line, picture). Then have them select the one that best presents and interprets (at a glance) the given data. Also, have them give their reasons for their selection.
7. Find the mean, median, and mode of a set of numbers; discuss their relative merits.	Take the class set of grades from the last test given. Use these to give an example problem to find mean, median, and mode. Have each student's height (can be measured quickly at beginning of period or last part of previous day's period) and use this as data to find mean, median, and mode. Also have students stand in descending heights. Put a mark on the wall for the tallest student, directly under for shortest student. Have someone measure to find distance halfway between. Then find the average and see if it is the same as the halfway mark. Have students bring examples to class of the use of mean, median, or mode in newspaper stories or other reports. Encourage discussion of which gives the most realistic representation for the context in which it is used. For example, how representative is the mean average salary in a discussion involving a very small company with a highly paid president?

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PROBABILITY AND STATISTICS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
8. Make predictions from given data.	Have students use an almanac or insurance reports to make a bar graph for automobile accidents in certain age groups 10 years ago and another bar graph for the current accident rate. Then lead students to make predictions for accident rates in certain age groups 10 years from now, based on present trends. Conduct other experiments, display on a line graph, and make predictions.	
***9. Make and interpret a scattergram for two sets of data.	Measure the heights of the students (in centimeters) and record their ages in months. Using the resulting ordered pairs (height, age) draw a scatter diagram. Have students analyze the scattergram. If there was a bunching at one corner of the diagram, how could it be redrawn to spread out the points? Do the points follow a pattern?	

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Unit Title: USING MATHEMATICS TO MEET PERSONAL/SOCIETAL NEEDS

Objectives	Suggested Teaching Activities	Resources/Comments
1. Make a budget for personal finances, including income from various sources; installment payments; basic necessities of food, clothing, shelter, and savings; and miscellaneous expenditures.	Have students participate in a simulation game where they draw a card to determine their incomes. Costs for basic necessities can be set according to prices in the present market. Students can make a budget to determine what installment payments and miscellaneous expenditures can be afforded. Include Windfall and Catastrophe cards to be drawn to represent unexpected income or expenses. Have students translate their budgeted amounts into percentages of income and compare to recommended budget allowances ($\frac{1}{4}$ monthly income for housing, etc.).	
2. Check a typical paycheck stub to determine gross amount, rates, tax deductions, FICA, insurance, dues, and net amount.	Discuss the meaning of each item on the paycheck stub. Which items are subtracted from the gross amount? Which items, if any, are added to the gross amount (bonuses, profit-sharing, etc.)? Which items are mandatory deductions and which are optional?	
3. Find the interest charges and the total amount of an installment loan.	Have students simulate purchases of automobiles, stereos, etc. Set up various lending institutions from which the students may choose to borrow: a bank requiring a 5% down payment with 12% interest, a credit union requiring a 10% down payment with 9.5% interest, a department store financing office with a revolving charge account, etc. Let students compare the various arrangements as to the advantages and disadvantages.	
4. Find sales tax, federal income tax, real property tax, personal property tax, etc.	Discuss the need for the various taxes and how they are set. Have students use their skills in finding percentages to determine tax amounts. Pose hypothetical situations in which a certain amount of money needs to be raised. Which tax rate(s) should be changed and by how much?	

USING MATHEMATICS TO MEET PERSONAL/SOCIETAL NEEDS, continued

Objectives	Suggested Teaching Activities	Resources/Comments
5. Reconcile a balance for a checking or savings account.	Discuss vocabulary, such as *deposit* and *withdrawal.* Simulation games may be used here also, or students may "earn" and "spend" points through specific class activities while maintaining a record of their deposits and withdrawals.	
6. Determine the role of mathematics in various careers and hobbies.	Discuss the role in society of each specific topic covered: probability in weather forecasting and advertising, statistics in sports and politics, and measurement procedures in science, engineering, manufacturing, carpentry, sewing, interior decorating, etc. Have students make a bulletin board display of situations involving mathematics. Encourage them to look in newspapers, magazines, brochures, instruction booklets. Change the material frequently to maintain interest and to show what an ever-present role mathematics plays in life.	

Unit Title: COMPUTER LITERACY

Objectives	Suggested Teaching Activities	Resources/Comments
1. Display a knowledge of the history of computer science and the role of computers and calculators in modern society.	Have students research the development of computers and report findings. Have students research the varied uses of computers and calculators and report findings. Bring community persons in to discuss the role of computers in their businesses or industries. Have students visit businesses, industries, or perhaps a university to observe computers in action.	
2. List the limitations and strengths of the computer by noting what it can and cannot do.	Emphasize to students that computers can only do things they have been programmed to do. They cannot think. Illustrate this concept by discussing the use of computer language. Emphasize that the strengths of computers are the speed at which they can do a great number of things, their ability to store data, and their accuracy. Ask students to list five things computers can do and five things computers cannot do.	

Appendices

Appendix A: *An Agenda for Action:*
Recommendations for School Mathematics of the 1980s

Appendix B: Correlation of TABS Objectives
To Mathematics Program Objectives

Appendix A

An Agenda for Action: Recommendations for School Mathematics of the 1980s*

The National Council of Teachers of Mathematics recommends that—

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of the mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism; and
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

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*National Council of Teachers of Mathematics, *An Agenda for Action: Recommendations for School Mathematics of the 1980s*. Reston, VA: NCTM, 1980.

Appendix B

Correlation of TABS Objectives to Mathematics Program Objectives

Exit Level Mathematics Objectives for TABS	6	7	8
1. Add and subtract whole numbers where regrouping is required.	3,4 (pp.17-18)	2 (p.56)	4(p.95) 6(p.96)
2. Multiply and divide whole numbers.	5(p.20) 8(p.21)	4,5 (p.57)	4(p.95) 7(p.96)
3. Solve problems using the basic operations.	7(p.18) 13(p.25) 8(p.37)	9 (p.58)	8,9 (p.96)
4. Add, subtract, and multiply fractions and mixed numbers.	1,3(p.30) 5(p.31) 9(p.33)	1-3(p.74) 5,6(p.76)	5(p.104) 6(p.105)
5. Add or subtract, multiply, and divide decimal fractions.	1-6 (pp.35-36)	2(p.56) 4,5(p.57)	4-7 (pp.95-96)
6. Solve problems involving income, banking, cost comparisons, and taxes.		6 (p.83)	1-6 (pp.127-128)
7. Determine total dollar amounts and/or correct change from a specified amount.			
8. Use the basic operations to solve problems involving measures.	12 (p.43)	26,27 (p.70)	3 (p.96)
9. Solve problems involving ratios, proportions, and percents.		5(p.81) 3-6(p.82)	3(p.106) 5(p.107)
10. Use maps to determine approximate distances or locations.		28(p.70) 2(p.90)	4 (p.116)
11. Read and interpret mathematical information from a chart or graph.	1(p.46) 3(p.47)	3 (p.91)	6 (p.125)

COMPLIANCE STATEMENT

TITLE VI, CIVIL RIGHTS ACT OF 1964; THE MODIFIED COURT ORDER, CIVIL ACTION 5281, FEDERAL DISTRICT COURT, EASTERN DISTRICT OF TEXAS, TYLER DIVISION

Reviews of local education agencies pertaining to compliance with Title VI Civil Rights Act of 1964 and with specific requirements of the Modified Court Order, Civil Action No. 5281, Federal District Court, Eastern District of Texas, Tyler Division are conducted periodically by staff representatives of the Texas Education Agency. These reviews cover at least the following policies and practices:

(1) acceptance policies on student transfers from other school districts;

(2) operation of school bus routes or runs on a non-segregated basis;

(3) nondiscrimination in extracurricular activities and the use of school facilities;

(4) nondiscriminatory practices in the hiring, assigning, promoting, paying, demoting, reassigning, or dismissing of faculty and staff members who work with children;

(5) enrollment and assignment of students without discrimination on the basis of race, color, or national origin;

(6) nondiscriminatory practices relating to the use of a student's first language; and

(7) evidence of published procedures for hearing complaints and grievances.

In addition to conducting reviews, the Texas Education Agency staff representatives check complaints of discrimination made by a citizen or citizens residing in a school district where it is alleged discriminatory practices have occurred or are occurring.

Where a violation of Title VI of the Civil Rights Act is found, the findings are reported to the Office for Civil Rights, Department of Health, Education and Welfare.

If there is a direct violation of the Court Order in Civil Action No. 5281 that cannot be cleared through negotiation, the sanctions required by the Court Order are applied.

TITLE VII, CIVIL RIGHTS ACT OF 1964; EXECUTIVE ORDERS 11246 AND 11375; TITLE IX, 1973 EDUCATION AMENDMENTS; REHABILITATION ACT OF 1973 AS AMENDED; 1974 AMENDMENTS TO THE WAGE-HOUR LAW EXPANDING THE AGE DISCRIMINATION IN EMPLOYMENT ACT OF 1967; AND VIETNAM ERA VETERANS READJUSTMENT ASSISTANCE ACT OF 1972 AS AMENDED IN 1974.

It is the policy of the Texas Education Agency to comply fully with the nondiscrimination provisions of all federal and state laws and regulations by assuring that no person shall be excluded from consideration for recruitment, selection, appointment, training, promotion, retention, or any other personnel action, or be denied any benefits or participation in any programs or activities which it operates on the grounds of race, religion, color, national origin, sex, handicap, age, or veteran status (except where age, sex, or handicap constitute a bona fide occupational qualification necessary to proper and efficient administration). The Texas Education Agency makes positive efforts to employ and advance in employment all protected groups.